

Math 102, Practice Test for Final Exam

Circle the correct answer for each problem.

**1. Factor.**

$$2a^2b + 2b^3 - 3a^3 - 3ab^2$$

**2. Factor.**

$$3x^2 - 16x - 12$$

**3. Factor.**

$$x^3 - 343$$

**4. Factor completely.**

$$10x^4 - 810$$

**5. Solve**

$$x^2 - x - 12 = 0$$

**6. Solve**

$$8x^2 + 34x + 35 = 0$$

**7. Determine the domain of this function.**

$$f(x) = \frac{x^2 - x - 6}{x^2 - 4}$$

**8. Simplify**

$$\frac{x+2}{x^2-4}$$

**9.** Simplify

$$\frac{1}{x} - \left( \frac{x^2}{3} \div \frac{2}{x} \right)$$

**10.** Simplify

$$\frac{\frac{2}{3}}{\frac{1}{3} + \frac{1}{x}}$$

**11.** Solve

$$\frac{1}{4} + \frac{3}{x} = \frac{1}{2}$$

**12.** Solve

$$\frac{1}{x+2} - \frac{1}{3} = \frac{1}{6}$$

**13.** What is the slope of the line with the equation

$$2x + 7y = 28$$

**14.** What is the equation of the line that passes through

$$(-5, 3) \text{ and } (2, -4)$$

**15.** Solve the system

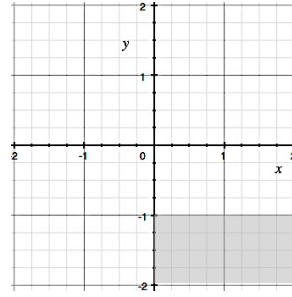
$$\begin{cases} 3x + y = 1 \\ 3x - 2y = 10 \end{cases}$$

**16.** Solve the system

$$\begin{cases} 10x + 2y = 2 \\ y = -5x + 1 \end{cases}$$

17. A friend invests \$1500 more at 6% than he invests at 8%. The interest from the two investments is the same. How much did he invested at 6%?

18. The shaded region is the graph of which system?



- (a)  $\begin{cases} x \leq 0 \\ y \leq -1 \end{cases}$       (b)  $\begin{cases} x \geq 0 \\ y \leq -1 \end{cases}$
- (c)  $\begin{cases} x \leq 0 \\ y \geq -1 \end{cases}$       (d)  $\begin{cases} x \geq 0 \\ y \geq -1 \end{cases}$

19. Simplify

$$\sqrt[4]{16y^4}$$

20. Simplify

$$\sqrt{27} + \sqrt{48}$$

**21.** Rationalize the denominator

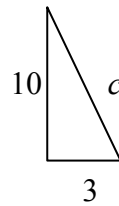
$$\frac{1}{5 - \sqrt{2}}$$

**22.** Rationalize the denominator

$$\frac{1}{4 + 3i}$$

**23.** Simplify

$$\sqrt[3]{\sqrt[5]{x^{60}}}$$

**24.** Find the length of the unknown side of the right triangle shown.

25. Simplify.

$$x^{\frac{2}{3}} \cdot x^{\frac{7}{3}}$$

26. Simplify.

$$\sqrt[4]{x} \cdot \sqrt{x}$$

27. Solve using the square root method.

$$(x - 2)^2 = 10$$

28. Solve by completing the square.

$$x^2 + 10x = -10$$

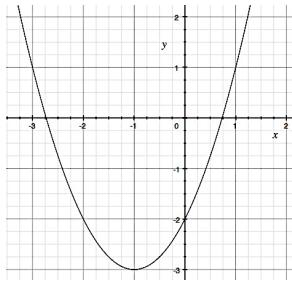
**29.** Solve by using the quadratic formula.

$$x^2 + 2x + 3 = 0$$

**30.** Solve.

$$x^2 - 3x - 10 > 0$$

**31.** Determine the equation of the parabola shown



(a)  $y = (x - 1)^2 - 3$

(b)  $y = (x - 3)^2 - 1$

(c)  $y = (x + 1)^2 - 3$

(d)  $y = (x - 1)^2 + 3$

**32.** Determine the vertex of the parabola.

$$y = x^2 - 10x + 2$$



**33.** The area of a rectangle is 198 square meters. Its length is five meters shorter than three times its width. What is the length?

**34.** Find the inverse function.

$$f(x) = \frac{3x + 3}{9}$$

**35.** Solve.

$$16^{x+2} = 8$$

**36.** Solve.

$$3^x = 12$$

**37. Solve.**

$$e^{-3t} = 15$$

**38. Solve.**

$$\log_6(x^2 - 6x + 15) = \log_6(5x - 13)$$

**39. Solve**

$$3\log x = 6$$

**40. Solve.**

$$\log_6(x) + \log_6(x + 1) = \log_6(12)$$

*For Reference*

Factoring Methods for Polynomials of 2-4 Terms						
Number of terms	Step 1	Identify the polynomial or polynomial factor	Step 2			
2	Factor out any GCF	Difference of squares $\Rightarrow$	$a^2 - b^2 = (a + b)(a - b)$			
		Sum of cubes $\Rightarrow$	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$			
		Difference of cubes $\Rightarrow$	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$			
		None of the above $\Rightarrow$	Prime – or cannot be factored by methods shown in this course.			
3	Factor out any GCF	$x^2 + bx + c \Rightarrow$	$= (x \quad)(x \quad)$ <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>Product <math>C</math></td></tr> <tr><td> </td></tr> <tr><td>Sum <math>b</math></td></tr> </table>	Product $C$		Sum $b$
		Product $C$				
Sum $b$						
$ax^2 + bx + c \Rightarrow$	$ax^2 + bx + c = ax^2 + b_1x + b_2x + c$ Then factor by grouping or box method. <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>Product <math>ac</math></td></tr> <tr><td><math>b_1</math>   <math>b_2</math></td></tr> <tr><td>Sum <math>b</math></td></tr> </table>	Product $ac$	$b_1$   $b_2$	Sum $b$		
Product $ac$						
$b_1$   $b_2$						
Sum $b$						
None of the above $\Rightarrow$	Prime – or cannot be factored by methods shown in this course.					
4	Factor out any GCF		Try to factor by grouping or box method.			

**Rules for Fractions** For any real numbers,  $a, b, c$ , and  $d, b \neq 0, c \neq 0$ , and  $d \neq 0$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c} \qquad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \qquad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Properties of Equality	Properties of Inequality			
For any real numbers, $a, b, c$ ,  If $a = b$ , then $a + c = b + c$  and $ac = bc$	For any real numbers, $a, b$ , and $c > 0$		For any real numbers, $a, b$ , and $c < 0$	
	If $a < b$ , then $a + c < b + c$  and $ac < bc$	If $a > b$ , then $a + c > b + c$  and $ac > bc$	If $a > b$ , then $a + c > b + c$  and $ac < bc$	If $a < b$ , then $a + c < b + c$  and $ac > bc$

Consistent systems		Inconsistent system
Independent equations	Dependent equations	
The two lines intersect in a single point.	The equations describe the same line.	The lines are parallel.
System solution set: $\{(2, -1)\}$	System solution set: $\{(x, y) \mid 2x + y = 3\}$	System solution set: $\emptyset$

Rules for Exponents	
Product Rule	$a^m \cdot a^n = a^{m+n}$
Quotient Rule	$\frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$
Power Rules	$(a^m)^n = a^{mn}$
	$(ab)^m = a^m b^m$
	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \quad (b \neq 0)$
Zero Exponent	$a^0 = 1 \quad (a \neq 0)$
Negative Exponent	$a^{-n} = \frac{1}{a^n} \quad (a \neq 0)$
Rational Exponent	$(a \geq 0 \text{ when } n \text{ is even})$

Rules for Radicals	
Definition	$\sqrt[n]{a^n} = a \quad (\text{for } a \geq 0 \text{ if } n \text{ is even})$
Product Rule	$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$
Quotient Rules	$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
Distributive Property	$a\sqrt{x} + b\sqrt{x} = (a+b)\sqrt{x}$
Definition	$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad (\text{for } a \geq 0 \text{ if } n \text{ is even})$

Square Root Property	
Let $x$ and $k$ be any two complex numbers.	<p>If <math>x^2 = k</math>,                      then <math>x = \sqrt{k}</math> or <math>x = -\sqrt{k}</math>.                      (We will use the notation <math>x = \pm\sqrt{k}</math> to represent <math>x = \sqrt{k}</math> or <math>x = -\sqrt{k}</math>.)</p>

Property for Solving an Exponential Equation
If $a^x = a^k$ , then $x = k$ (for $a > 0$ and $a \neq 1$ ).

General Principles of the Vertical Parabola	
$y = a(x - h)^2 + k, a \neq 0$	
Vertex	$(h, k)$
Axis of symmetry	$x = h$
Opens up	$a > 0$
Opens down	$a < 0$
Wider than $y = x^2$	$0 <  a  < 1$
Narrower than $y = x^2$	$ a  > 1$

Properties of Logarithms	
If $x, y$ , and $b$ are positive real numbers, where $b \neq 1$ , and $r$ is any real number, then the following are true:	
Definition	$\log_b x = y \Leftrightarrow b^y = x$ ( $x > 0, b > 0$ and $b \neq 1$ )
Basic Properties	$\log_b b = 1$ and $\log_b 1 = 0$
Product Rule	$\log_b xy = \log_b x + \log_b y$
Quotient Rule	$\log_b \frac{x}{y} = \log_b x - \log_b y$
Power Rule	$\log_b x^r = r \log_b x$
Special Properties	$b^{\log_b x} = x$ and $\log_b b^x = x$
Logarithm Property of Equality	$\log_b x = \log_b y \Leftrightarrow x = y$ ( $x > 0, y > 0, b > 0$ and $b \neq 1$ )