Math 102, Practice Test for Final Exam

Circle the correct answer for each problem.

1. Factor. $2a^2b + 2b^3 - 3a^3 - 3ab^2$	**2.** Factor. $3x^2 - 16x - 12$
3. Factor. $x^3 - 343$	**4.** Factor completely. $10x^4 - 810$

5. Solve $x^2 - x - 12 = 0$	**6.** Solve $8x^2 + 34x + 35 = 0$
7. Determine the domain of this function. $f(x) = \frac{x^2 - x - 6}{x^2 - 4}$	**8.** Simplify $\frac{x+2}{x^2 - 4}$

9. Simplify

$$\frac{1}{x}-\left(\frac{x^2}{3}\div\frac{2}{x}\right)$$

10. Simplify

$$\frac{\frac{2}{3}}{\frac{1}{3}+\frac{1}{x}}$$

11. Solve

$$\frac{1}{4}+\frac{3}{x}=\frac{1}{2}$$

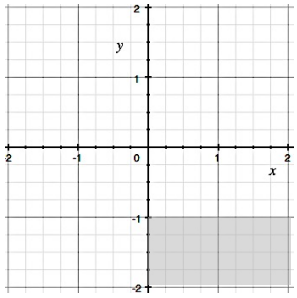
12. Solve

$$\frac{1}{x+2}-\frac{1}{3}=\frac{1}{6}$$

13. What is the slope of the line with the equation $2x + 7y = 28$	**14.** What is the equation of the line that passes through $(-5, 3)$ and $(2, -4)$
15. Solve the system $\begin{cases} 3x + y = 1 \\ 3x - 2y = 10 \end{cases}$	**16.** Solve the system $\begin{cases} 10x + 2y = 2 \\ y = -5x + 1 \end{cases}$

17. A friend invests \$1500 more at 6% than he invests at 8%. The interest from the two investments is the same. How much did he invested at 6%?

18. The shaded region is the graph of which system?



(a) $\begin{cases} x \le 0 \\ y \le -1 \end{cases}$ (b) $\begin{cases} x \ge 0 \\ y \le -1 \end{cases}$

(c) $\begin{cases} x \le 0 \\ y \ge -1 \end{cases}$ (d) $\begin{cases} x \ge 0 \\ y \ge -1 \end{cases}$

19. Simplify

$$\sqrt[4]{16y^4}$$

20. Simplify

$$\sqrt{27} + \sqrt{48}$$

21. Rationalize the denominator

$$\frac{1}{5-\sqrt{2}}$$

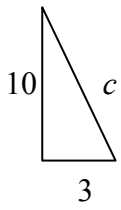
22. Rationalize the denominator

$$\frac{1}{4+3i}$$

23. Simplify

$$\sqrt[3]{\sqrt[5]{x^{60}}}$$

24. Find the length of the unknown side of the right triangle shown.



25. Simplify. $x^{\frac{2}{3}} \bullet x^{\frac{7}{3}}$	**26.** Simplify. $\sqrt[4]{x} \bullet \sqrt{x}$
27. Solve using the square root method. $(x-2)^2 = 10$	**28.** Solve by completing the square. $x^2 + 10x = -10$

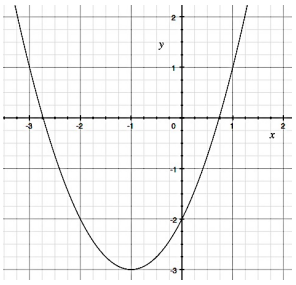
29. Solve by using the quadratic formula.

$$x^2 + 2x + 3 = 0$$

30. Solve.

$$x^2 - 3x - 10 > 0$$

31. Determine the equation of the parabola shown



(a) $y = (x - 1)^2 - 3$

(b) $y = (x - 3)^2 - 1$

(c) $y = (x + 1)^2 - 3$

(d) $y = (x - 1)^2 + 3$

32. Determine the vertex of the parabola.

$$y = x^2 - 10x + 2$$

33. The area of a rectangle is 198 square meters. Its length is five meters shorter than three times its width. What is the length?

34. Find the inverse function.

$$f(x) = \frac{3x+3}{9}$$

35. Solve.

$$16^{x+2} = 8$$

36. Solve.

$$3^x = 12$$

37. Solve. $e^{-3t} = 15$	**38.** Solve. $\log_6(x^2 - 6x + 15) = \log_6(5x - 13)$
39. Solve $3\log x = 6$	**40.** Solve. $\log_6(x) + \log_6(x + 1) = \log_6(12)$

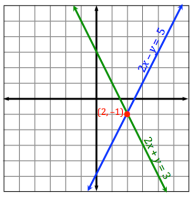
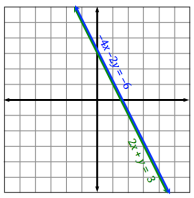
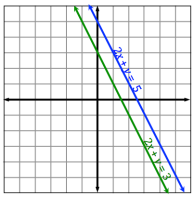
For Reference

Factoring Methods for Polynomials of 2-4 Terms			
Number of terms	*Step 1*	*Identify the polynomial or polynomial factor*	*Step 2*
2	Factor out any GCF	Difference of squares $\Rightarrow$	$a^2 - b^2 = (a + b)(a - b)$
		Sum of cubes $\Rightarrow$	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
		Difference of cubes $\Rightarrow$	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
		None of the above $\Rightarrow$	Prime – or cannot be factored by methods shown in this course.
3	Factor out any GCF	$x^2 + bx + c \Rightarrow$	$= (x \quad)(x \quad)$ Product c Sum b
		$ax^2 + bx + c \Rightarrow$	$ax^2 + bx + c$ $= ax^2 + b_1x + b_2x + c$ Then factor by grouping or box method. Product ac b_1 | b_2 Sum b
		None of the above $\Rightarrow$	Prime – or cannot be factored by methods shown in this course.
4	Factor out any GCF		Try to factor by grouping or box method.

Rules for Fractions For any real numbers, a, b, c, and d, $b \neq 0$, $c \neq 0$, and $d \neq 0$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c} \qquad \frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd} \qquad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$$

Properties of Equality	**Properties of Inequality**			
For any real numbers, a, b, c,	For any real numbers, a, b, and $c > 0$		For any real numbers, a, b, and $c < 0$	
If $a = b$, then $a + c = b + c$ and $ac = bc$	If $a < b$, then $a + c < b + c$ and $ac < bc$	If $a > b$, then $a + c > b + c$ and $ac > bc$	If $a > b$, then $a + c > b + c$ and $ac < bc$	If $a < b$, then $a + c < b + c$ and $ac > bc$

Consistent systems		**Inconsistent system**
Independent equations	**Dependent equations**	
		
The two lines intersect in a single point.	*The equations describe the same line.*	*The lines are parallel.*
System solution set: $\{(2, -1)\}$	System solution set: $\{(x, y) \mid 2x + y = 3\}$	System solution set: $\varnothing$

Rules for Exponents	
Product Rule	$a^m \cdot a^n = a^{m+n}$
Quotient Rule	$\frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$
Power Rules	$(a^m)^n = a^{mn}$ $(ab)^m = a^m b^m$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \quad (b \neq 0)$
Zero Exponent	$a^0 = 1 \quad (a \neq 0)$
Negative Exponent	$a^{-n} = \frac{1}{a^n} \quad (a \neq 0)$
Rational Exponent	($a \geq 0$ when n is even)

Rules for Radicals	
Definition	$\sqrt[n]{a^n} = a$ (for $a \geq 0$ if n is even)
Product Rule	$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$
Quotient Rules	$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
Distributive Property	$a\sqrt{x} + b\sqrt{x} = (a+b)\sqrt{x}$
Definition	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$ (for $a \geq 0$ if n is even)

Square Root Property	
Let x and k be any two complex numbers.	If $x^2 = k$, then $x = \sqrt{k}$ or $x = -\sqrt{k}$. (We will use the notation $x = \pm\sqrt{k}$ to represent $x = \sqrt{k}$ or $x = -\sqrt{k}$.)

Property for Solving an Exponential Equation
If $a^x = a^k$, then $x = k$ (for $a > 0$ and $a \neq 1$).

General Principles of the Vertical Parabola	
$y = a(x-h)^2 + k, a \neq 0$	
Vertex	(h, k)
Axis of symmetry	$x = h$
Opens up	$a > 0$
Opens down	$a < 0$
Wider than $y = x^2$	$0 < \lvert a \rvert < 1$
Narrower than $y = x^2$	$\lvert a \rvert > 1$

Properties of Logarithms	
If x, y, and b are positive real numbers, where $b \neq 1$, and r is any real number, then the following are true:	
Definition	$\log_b x = y \Leftrightarrow b^y = x$ ($x > 0$, $b > 0$ and $b \neq 1$)
Basic Properties	$\log_b b = 1$ and $\log_b 1 = 0$
Product Rule	$\log_b xy = \log_b x + \log_b y$
Quotient Rule	$\log_b \frac{x}{y} = \log_b x - \log_b y$
Power Rule	$\log_b x^r = r \log_b x$
Special Properties	$b^{\log_b x} = x$ and $\log_b b^x = x$
Logarithm Property of Equality	$\log_b x = \log_b y \Leftrightarrow x = y$ ($x > 0$, $y > 0$, $b > 0$ and $b \neq 1$)