1. Show that in a metric space (X, d), $\lim_{n\to\infty} x_n = x \Leftrightarrow \lim_{n\to\infty} d(x_n, x) = 0$.

2. Let $\{x_n\}$, $\{y_n\}$, $\{z_n\}$ be sequences of real numbers. Suppose that $x_n \le y_n \le z_n$, $\forall\, n \in \mathbb{N}$ and that $x_n \to \alpha$ and $z_n \to \alpha$. Show that $y_n \to \alpha$ as $n \to \infty$.

3. Let $\{x_n\} \subset X$ and $\{\varepsilon_n\} \subset \mathbb{R}$, $\varepsilon_n \ge 0$, $\forall\, n \in \mathbb{N}$. Suppose that $\lim_{n\to\infty} \varepsilon_n = 0$ and $d(x_n, x) \le \varepsilon_n$,

4. Let $\{x_n\}$, $\{y_n\} \subset X$. Suppose that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} d(x_n, y_n) = 0$. Show that $\lim_{n\to\infty} y_n = x$.

5. Show that if $\{x_n\} \subset X$ has two subsequences that converge to different limits, then $\{x_n\}$ is divergent.

6. Let $\{x_n\} \subset \mathbb{R}$ such that $x_n > 0$, $\forall\, n \in \mathbb{N}$ and $\lim_{n\to\infty} \frac{x_{n+1}}{x_n} = L$. Show that if $L < 1$ then $\lim_{n\to\infty} x_n = 0$. Show that if $L > 1$ then $x_n \to +\infty$. What about the case $L = 1$?

7. Let $0 \le a \le b$. Show that $\sqrt[n]{a^n + b^n} \to b$.

8. Let $\{x_n\} \subset \mathbb{R}^n$. Show that if $x_n \to x$ then $|x_n| \to |x|$. Does $|x_n| \to |x|$ imply $x_n \to x$? Show that if $|x_n| \to 0$ then $x_n \to 0$.

9. Let $\{x_n\}$ be such that $x_n \to +\infty$. Show that $\lim_{n\to\infty} \frac{1}{x_n} = 0$.

10. Let $\{x_n\} \subset \mathbb{R}$ such that $\{x_{2n}\}$ and $\{x_{2n+1}\}$ converge to the same number. Show that $\{x_n\}$ is convergent.

11. Let $\{x_n\}$ and $\{y_n\}$ be real sequences. Suppose that $\{x_n\}$ converges to a non-zero number and that $\{x_n \bullet y_n\}$ is convergent. Show that $\{y_n\}$ is convergent too.
What if $x_n \to 0$?