2. Use the sequence of partial sums to decide whether the following series converge or diverge:

(a) $\sum_{k=1}^{\infty}(-1)^{k+1}$;

(b) $\sum_{n=1}^{\infty}\frac{2^n}{3^{n+1}}$; .

(d) $\sum_{n=1}^{\infty}\frac{1}{n(n+2)}$;

(e) $\sum_{k=3}^{\infty}\frac{1}{k^2-4}$;

(f) $\sum_{k=1}^{\infty}\frac{1}{\sqrt{k+1}\sqrt{k}}$;

3. Show that $\sum_{n=1}^{\infty}\left(a_n - a_{n+1}\right)$ converges $\Leftrightarrow$ the sequence $\{a_n\}$ converges.

4. Let $-1 < a < 1$. Prove that $\sum_{n=1}^{\infty}\frac{1}{1+a^n}$ diverges.

5. Show that $\sum_{k=1}^{\infty}\sin k$ diverges.

6. Let $p > 1$. Show that $\sum_{n=1}^{\infty}\frac{1}{n^p} < \frac{2^p}{2^p-2}$.

7. Use comparison tests to determine whether or not $\sum_{n=1}^{\infty}a_n$ converges if a_n is given by

(a) $a_n = \frac{1}{n^2+2n-1}$;

(b) $a_n = \frac{1}{\sqrt{n^3+5}}$;

(c) $a_n = \frac{1}{\ln(n+1)}$.

8. Suppose that $\sum_{k=1}^{\infty}a_k$ converges absolutely. Show that $\sum_{k=1}^{\infty}a_k^2$ converges. Also, show that it is not enough to assume just that $\sum_{k=1}^{\infty}a_k$ converges.

9. Let $a_n = \begin{cases}\frac{1}{n^2} \text{ if } n \neq 2^k \\ 1 \text{ if } n = 2^k\end{cases}$, where $k \in \mathbb{N}$. Show that $\sum_{n=1}^{\infty}\frac{1}{1+na_n}$ converges.

10. Let $\sum_{n=1}^{\infty} a_n$ be a series with positive terms and suppose that $a_n \neq 1$, $\forall\, n \in \mathbb{N}$. Prove that:

(a) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \frac{a_n}{1 - a_n}$ converges.

(b) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{n}$ converges.

11. Suppose that $\lim_{n \to \infty} a_n = a \neq 0$ and $a_n \neq 0$ $\forall\, n \in \mathbb{N}$. Show that the 2 series $\sum_{n=1}^{\infty} |a_{n+1} - a_n|$ and $\sum_{n=1}^{\infty} \left| \frac{1}{a_{n+1}} - \frac{1}{a_n} \right|$ either both converge or both diverge.

12. Suppose $a_n > 0$ and $a_{n+1} \leq a_n$, $\forall\, n \in \mathbb{N}$. Prove that if $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n \to \infty} n a_n = 0$.