1. Determine whether the following series are absolutely convergent, conditionally convergent or divergent:

(a) $\sum_{k=1}^{\infty} \frac{\sin k}{2^k}$;

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2 2^n}{n!}$;

(c) $\sum_{n=1}^{\infty} n\left(\frac{2}{3}\right)^n$;

(d) $\sum_{n=1}^{\infty} \frac{n!}{10^n}$;

(e) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n+1}}$;

2. Consider the following real power series. Determine the radius of convergence and interval of convergence.

(a) $\sum_{n=1}^{\infty} \frac{5^n x^n}{n^2}$;

(b) $\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt[4]{n}}$.

3. Consider the following complex power series. Determine the radius of convergence and domain of convergence.

(a) $\sum_{n=1}^{\infty} \frac{n}{4^n}(z+i)^n$;

(b) $\sum_{n=1}^{\infty} \frac{z^n}{n^n}$.

4. What do the Root and Ratio tests tell about the convergence of the following series.

(a) $\sum_{n=1}^{\infty} \left(2+(-1)^n\right)2^{-n}$;

4. (b) $\sum_{n=1}^{\infty} \frac{1}{\left[3+(-1)^n\right]^n}$.

5. Let $\{s_n\}$ be the sequence of partial sums of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$.

Let $\{t_n\}$ be the sequence of partial sums of the rearranged series:

$1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}+\frac{1}{5}-\frac{1}{10}-\frac{1}{12}+\cdots$.

Show that $t_{3n} = \frac{1}{2}s_{2n}$. What can you say about the convergence of the two series?

6. Use the Cauchy product of the series $\sum_{n=0}^{\infty} \frac{a^n}{n!}$ and $\sum_{n=0}^{\infty} \frac{b^n}{n!}$ to show that $e^a \bullet e^b = e^{a+b}$.

7. Calculate the Cauchy product of the following divergent series: $\sum_{n=0}^{\infty} 2^n$ and $\sum_{n=0}^{\infty} (-1)^{n+1}$.

8. Let $\sum_{k=0}^{\infty} a_k$ be a series of real numbers such that $\sum_{k=1}^{\infty} a_k b_k$ converges for every bounded sequence $\{b_k\}$. Prove that $\sum_{k=1}^{\infty} a_k$ converges absolutely.