

## Math 230A, Week #1 - Practice Problems

1. Let  $A \subset \mathbb{R}$  be nonempty and bounded above. Show that if  $\alpha = \sup A \notin A$ , then  $\forall \varepsilon > 0$  the interval  $(\alpha - \varepsilon, \alpha)$  contains infinitely many elements of  $A$ .

What about the case  $\alpha \in A$ ?

2. Let  $A, B \subset \mathbb{R}$  be bounded sets. Show that

**a)**  $\inf(A + B) = \inf A + \inf B$

**b)**  $\sup(A + B) = \sup A + \sup B$

**c)** Is  $\sup(A \cdot B) = \sup A \cdot \sup B$ ?

**d)**  $\sup(-A) = -\inf A$

**e)**  $\sup(A - B) = \sup A - \inf B$

**f)**  $\sup(2A) = 2\sup A$

**g)**  $\inf A \leq \sup A$

**g)**  $\inf A$  and  $\sup A$  are unique. In other words every set  $A$  has at most one glb or lub.

**h)**  $\inf(A \cup B) = \min \{\inf A, \inf B\}$

**i)**  $\sup(A \cup B) = \max\{\sup A, \sup B\}$

What about  $A \cap B$ ?