HW02, Practice Problems, Assigned Thursday, 9/9/10

1. Read and understand Proposition 1.18 and its proof.
Proposition 1.18 The following statements are true in every ordered field.
(a) If $x > 0$, then $-x < 0$, and vice versa.
(b) If $x > 0$ and $y < z$, then $xy < xz$.
(c) If $x < 0$ and $y < z$, then $xy > xz$.
(d) If $x \neq 0$ then $x^2 > 0$. In particular, $1 > 0$.
(e) If $0 < x < y$ then $0 < 1/y < 1/x$.

2. Read Definition 1.24 of the field of Complex Numbers $\mathbb{C}$.
(a) Ex. 8, Ch. 1 says that no order can be defined which turn $\mathbb{C}$ into an ordered field. Explain.
(b) Consider the following partial order on $\mathbb{C}$: $(a, b) \leq (c, d)$ if $a \leq c$ and $b \leq d$.
What is the set of positive elements?
Does this order satisfy the two axioms of Definition 1.17

Hint: For exercises 3, 4 you may use the prime factorization theorem.
3. Let n be a positive integer which is not a perfect square. Prove that $\sqrt{n}$ is irrational.

Proof:

4. Prove that there is no rational number r such that $2^r = 3$.

5. Solve Exercise 12, Chapter 1.
If $z_1, \ldots, z_n$ are complex, prove that $|z_1 + z_2 + \cdots + z_n| \leq |z_1| + |z_2| + \cdots + |z_n|$.

6. Solve Exercise 13, Chapter 1.
If x, y are complex, prove that $||x| - |y|| \leq |x - y|$.

7. Solve Exercise 17, Chapter 1 in the case of complex numbers.
Prove that $|x + y|^2 + |x - y|^2 = 2|x|^2 + 2|y|^2$ if $x \in \mathbb{C}$ and $y \in \mathbb{C}$.
Interpret this geometrically, as a statement about parallelograms.

8. Prove Bernoulli's inequaltiy:
If $x > -1$ and $x \neq 0$, then $(1 + x)^n > 1 + nx$ for each $n \geq 2$.