

Pages 2-3, Assigned Tuesday 9/21/10

Consider non-empty sets A, B and a function $f: A \rightarrow B$.

Also, consider the subsets $A_1, A_2 \subset A$ and $B_1, B_2 \subset B$.

Use the definitions $f(A_1) = \{f(a) \in B: a \in A_1\}$ and $f^{-1}(B_1) = \{a \in A: f(a) \in B_1\}$.

1. Show that:

(a) $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$.

(b) $f(A_1 \cap A_2) \subset f(A_1) \cap f(A_2)$. Show an example for $f(A_1 \cap A_2) \neq f(A_1) \cap f(A_2)$.

(c) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$.

(d) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$.

(e) $f^{-1}(B \setminus B_1) = A \setminus f^{-1}(B_1)$.

2. This exercise regards the 1-1 functions:

(a) Show that if f is 1-1, then $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$.

(b) If $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$, for all $A_1, A_2 \subset A$, then f is 1-1.

(c) f is 1-1 $\Leftrightarrow f^{-1}(f(A_1)) = A_1$ for every $A_1 \subset A$.

(d) From the proof of (c), conclude that $A_1 \subset f^{-1}(f(A_1))$ for any function f .

(e) Show that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are 1-1, then $g \circ f: A \rightarrow C$ is 1-1.

3. This exercise regards the onto functions.

(a) Show that $f(f^{-1}(B_1)) \subset B_1$ for any function.

(b) f is onto $\Leftrightarrow f(f^{-1}(B_1)) = B_1$ for all $B_1 \subset B$.

(c) Show that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are onto functions, then $g \circ f: A \rightarrow C$ is 1-1 is onto.

4. This exercise regards the 1-1 and onto functions.

(a) Show that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are 1-1 and onto functions, then $g \circ f: A \rightarrow C$ is 1-1 and onto, and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

5. Think about the polynomial, rational, exponential, logarithmic, trigonometric functions. Examine their 1-1 and onto properties.

Polynomial Functions: Polynomials of even degree are not 1-1 or onto.

Polynomials of odd degree that are factorable over the reals are not 1-1, but are onto.

Exponential functions: 1-1, but not onto.

Logarithmic functions: 1-1, but the domain is not defined on all of \mathbb{R} .

Trigonometric functions:

Sine: Not 1-1 and not onto.

Cosine: Not 1-1 and not onto.

Tangent: 1-1 and onto, but not defined on all of \mathbb{R} .