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Chapter 5

Factoring

Lecture Note-Taking Guide

Scoring Rubric

Extra Credit Points	Condition	
5	• complete • all worksteps shown • correct answers • neatly done in pencil • correctly ordered	• hole punched • fastened securely in folders with fasteners • turned in on time • labeled with name and site
0	Any incomplete problems, worksteps missing, incorrect answers, illegible writing, incorrectly ordered pages, pages not fastened in folder with fasteners, or turned in late	

5.1 Greatest Common Factor

Prime Factorization

The result of a multiplication problem is called a *product*. The numbers being multiplied are called *factors*.

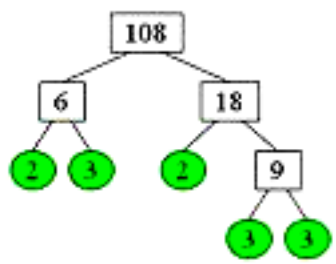
$$\underbrace{5 \times 6}_{\text{factors}} = \underbrace{30}_{\text{product}} \qquad \underbrace{x(x-3)}_{\text{factors}} = \underbrace{x^2 - 3x}_{\text{product}}$$

The *prime factorization* of a positive integer is the factorization in which all the factors are prime numbers. A few examples follow:

Number	*Factorization*	*Prime Factorization*
30	$6 \cdot 5$	$2 \cdot 3 \cdot 5$
4	2^2	2^2
36	$4 \cdot 9$	$2^2 \cdot 3^2$

Suppose we want to find the prime factorization of 108.

We can find prime factorizations by writing the number as a product of two factors, then writing each factor as a product of two factors, repeating this process until all the factors are prime numbers. A *factor tree* is useful in organizing this work as shown at right.



From the "leaves" of the factor tree, we have that the prime factorization of 108 is $2^2 \times 3^3$.

Or, alternatively, divide successively by prime factors as shown:

$$\begin{array}{r|l} 2 & \underline{108} \\ 2 & \underline{54} \\ 3 & \underline{27} \\ 3 & \underline{9} \\ & 3 \end{array}$$

$$108 = 2^2 \times 3^3$$

Demonstration Problems	*Practice Problems*
Write the prime factorization of	Write the prime factorization of
1. (*a*) 20	**1. (*b*)** 18
2. (*a*) 248	**2. (*b*)** 216

Answers: **1. (*b*)** $2 \cdot 3^2$; **2. (*b*)** $2^3 \cdot 3^3$

Greatest Common Factor

A *common factor* of two algebraic expressions is a factor that is common to both expressions. Consider the following examples:

Example (*a*) Find a common factor of 30 and 25.

$30 = 2 \cdot 3 \cdot 5 \qquad 25 = 5 \cdot 5$

5

is a common factor of 30 and 25

Example (*b*) Find a common factor of $2x^2y$ and $5yz$.

$2x^2y \qquad 5yz$

y

is a common factor of $2x^2y$ and $5yz$

The *greatest common factor*, or GCF, is the product of all common numerical and variable factors of two or more expressions. The GCF will be comprised of the product of the lowest degree of each common factor. Consider the following examples:

Example (*c*) Find the greatest common factor of 60 and 36.

60 and 36

$60 = 2^2 \cdot 3 \cdot 5 \quad 36 = 2^2 \cdot 3 \cdot 3$

$2^2 \cdot 3 = 12$

GCF (60, 36) = 12

Example (*d*) Find the greatest common factor of $3x^4y$ and $4x^3y$.

$3x^4y$ and $4x^3y$

$3x^4y = 3x \cdot x^3y \qquad 4x^3y$

x^3y

$\text{GCF}\ (3x^4y, 4x^3y) = x^3y$

Demonstration Problems	*Practice Problems*
3. (*a*) GCF (8, 20)	**3. (*b*)** GCF (18, 42)
4. (*a*) GCF ($3x^2y$, $6xy^2$)	**4. (*b*)** GCF ($2a^2x^3$, $12a^3x$)
	Answers: **3. (*b*)** 6; **4. (*b*)** $2a^2x$

Factoring Out the Greatest Common Factor

The Distributive Property gives us that for any real numbers a, b, and c,

$$a(b + c) = ab + ac.$$

If we start with $ab + ac$ and write it as a product of factors, $a(b + c)$, then the process is called *factoring*.

For example, $12x^3 - 8xy = 4x \cdot 3x^2 - 4x \cdot 2y = 4x(3x^2 - 2y)$

$$\text{GCF}\ (12x^3, 8xy) = 4x$$

Demonstration Problems	*Practice Problems*
Factor out the GCF in each expression.	Factor out the GCF in each expression.
5. (*a*) $2w + 4z$	**5. (*b*)** $6y + 3$
6. (*a*) $-2k^7m^4 + 4k^3m^6$	**6. (*b*)** $-6h^5j^2 + 3h^3j^6$
7. (*a*) $8a^2b^2 + 16a^2b - 24ab$	**7. (*b*)** $4a^3b^2 + 8a^2b^2 - 24ab^2$
8. (*a*) $a(x - 3) + b(x - 3)$	**8. (*b*)** $z(y + 4) + 3(y + 4)$

Answers: **5. (*b*)** $3(2y + 1)$; **6. (*b*)** $3h^3j^2(-2h^2 + j^4)$; **7. (*b*)** $4ab^2(a^2 + 2a - 6)$ **8. (*b*)** $(y + 4)(z + 3)$

5.2 Factoring by Grouping

In Chapter 4, we learned that two binomials can be multiplied by using the distributive property twice. And sometimes it is also taught that binomials can be multiplied using the box method. Let's review these techniques as we multiply the following:

$$\begin{aligned}&(x+y)(a+3)\\ &= x(a+3)+y(a+3)\\ &= ax+3x+ay+3y\end{aligned} \qquad \begin{aligned}&(x-y)(c-2)\\ &= x(c-2)-y(c-2)\\ &= cx-2x-cy+2y\end{aligned}$$

or alternatively, we could use the box method as follows:

	x	y
a	ax	ay
3	$3x$	$3y$

	x	$-y$
c	cx	$-cy$
-2	$-2x$	$2y$

Factoring by Grouping

Polynomials with four terms can sometimes be factored into a product of two binomials by reversing the process of multiplying.

Step 1: Factor the first group of two terms.
Step 2: Factor the second group of two terms.
Step 3: Factor the common binomial.

Example:
Factor $xy + 2y + 5x + 10$.

$$\begin{aligned}&xy+2y+5x+10\\ &= xy+2y+5x+5\cdot 2\\ &= y(x+2)+5(x+2)\\ &= (x+2)(y+5)\end{aligned}$$

Demonstration Problems	*Practice Problems*
Factor by grouping. **1. (*a*)** $bx + by + cx + cy$	Factor by grouping. **1. (*b*)** $3x + 3z + ax + az$
2. (*a*) $x^2 + 5x + 4x + 20$	**2. (*b*)** $y^2 + 2y + 6y + 12$

Answers: **1. (*b*)** $(3 + a)(x + z)$; **2. (*b*)** $(y + 6)(y + 2)$

Box Method Factoring

Alternatively, we can factor $xy + 2y + 5x + 10$ by the "box method".

Step 1: Draw a 2×2 box.
Step 2: Place the terms in the box in the order given.
Step 3: Factor the GCF of each row and column, placing the GCF to the left of or above the box.
Step 4: Read the factors from the left side and top side of the box.

Demonstration Problems	*Practice Problems*
Use the box method to factor. **3. (*a*)** $ap + 3a - p - 3$	Use the box method to factor. **3. (*b*)** $w^2 + aw - w - a$
4. (*a*) $2y^2 + 3y - 16y - 24$	**4. (*b*)** $2x^2 - 3x - 2x + 3$
	Answers: **3. (*b*)** $(w - 1)(w + a)$; **4. (*b*)** $(x - 1)(2x - 3)$

Note that the terms may need to be re-ordered before the preceding methods will work. For example:

Factor $ab + 12 + 6a + 2b$.

In the order given, the GCF of the top row is 1 and the GCF of the first column is 1, but $1 \cdot 1 \neq ab$.

	?	
?	ab	12
	$6a$	$2b$

If we change the order so that the GCF of the top row and the first column is not 1, then the method yields the proper binomial factors.

	a	2
b	ab	$2b$
6	$6a$	12

5.3 Factoring Trinomials

Factoring the Trinomial $ax^2 + bx + c$ with $a = 1$

Notice the following 3 multiplying and simplifying patterns:

1. $(x + 4)(x + 3)$

$x^2 + 3x + 4x + 12$

$x^2 + 7x + 12$

2. $(x + 4)(x + 3)$

$x^2 + 3x + 4x + 12$

$x^2 + 7x + 12$

3. $(x + 4)(x + 3)$

$x^2 + 3x + 4x + 12$

$x^2 + 7x + 12$

To factor the trinomial $x^2 + 7x + 12$ into the product of two binomials, without already knowing the binomials, we need to get "clues" from the 3 terms.

From (**1**), we see x^2 resulted from binomials starting with $(x + \_\_\_\_)(x + \_\_\_\_)$.

From (**2**), we know that 12 is a product of the two constants in the binomials. But this is not a strong enough clue on its own to complete the factorization that we started from pattern (**1**). There are many pairs of factors whose product is 12.

$1 \bullet 12 = 12$		$-1 \bullet -12 = 12$
$2 \bullet 6 = 12$	and	$-2 \bullet -6 = 12$
$3 \bullet 4 = 12$		$-3 \bullet -4 = 12$

From (**3**), we know that 7 is the sum of the two constants in the binomials. Since 12 is the product of the same two numbers, we can use the list of product pairs to find two numbers whose product is **12** AND whose sum is **7**.

Product is 12	**Sum is 7**
$1 \bullet 12 = 12$	$1 + 12 = 13$
$2 \bullet 6 = 12$	$2 + 6 = 8$
$\mathbf{3 \bullet 4 = 12}$	$\mathbf{3 + 4 = 7}$
$-1 \bullet -12 = 12$	$-1 + -12 = -13$
$-2 \bullet -6 = 12$	$-2 + -6 = -8$
$-3 \bullet -4 = 12$	$-3 + -4 = -7$

To save effort, a shorthand list could be created as follows:

Product 12	
1	12
2	6
3	**4**
Sum 7	

Mentally check the sum of each pair until the desired sum is found.

We see that: $3 \bullet 4 = 12$ and $3 + 4 = 7$

So then

$$x^2 + 7x + 12 = (x + \underline{\ 3\ })(x + \underline{\ 4\ }).$$

Demonstration Problems	*Practice Problems*
Factor the following trinomials.	Factor the following trinomials.
1. *(a)* $a^2 + 9a + 20$	**1. *(b)*** $x^2 + 12x + 20$
2. *(a)* $x^2 + 2x - 24$	**2. *(b)*** $b^2 - 7b + 10$
3. *(a)* Factor $t^2 + 3t - 5$.	**3. *(b)*** $r^2 + 4r + 7$

1. *(a)* $a^2 + 9a + 20$

$= (a \qquad)(a \qquad)$

Product 20	
1	
2	
Sum 9	

2. *(a)* $x^2 + 2x - 24$

$= (x \qquad)(x \qquad)$

Product −24	
−1	
−2	
Sum 2	

3. *(a)* Factor $t^2 + 3t - 5$.

$= (t \qquad)(t \qquad)$

Product −5	
−1	
Sum 3	

Answers: **1. *(b)*** $(x + 2)(x + 10)$; **2. *(b)*** $(b - 5)(b - 2)$; **3. *(b)*** prime

Consider another pattern:

$$(x + 4y)(x + 3y)$$

$$x^2 + 3xy + 4xy + 12y^2$$

$$x^2 + 7xy + 12y^2$$

From this, we can see x^2 and _____y^2 resulted from binomials containing

$$(x + \_\_\_\_ y)(x + \_\_\_\_ y).$$

To complete the factoring of $x^2 + 7xy + 12y^2$, we follow the same procedure as we did in the above pages by finding two numbers whose product is 12 and sum is 7 and inserting those numbers before the y in each binomial.

Demonstration Problems	*Practice Problems*
Factor each trinomial. **4. (*a*)** $x^2 + 2xy - 8y^2$ $= (x \_\_\_\_\_ y)(x \_\_\_\_\_ y)$ Product -8 -1 -2 Sum 2	Factor each trinomial. **4. (*b*)** $a^2 - 7ab + 10b^2$
	Answer: **4. (*b*)** $(a - 2b)(a - 5b)$

To factor completely, first look for a GCF, factor it out if there is one, then factor the resulting polynomial if it can be factored.

Demonstration Problems	*Practice Problems*
Factor completely. **5. (*a*)** $6w^2 - 12w - 18$ $= 6(w^2 - 2w - 3)$ $= 6(w \qquad)(w \qquad)$ Product -3 -1 Sum -2	Factor completely. **5. (*b*)** $2w^2 - 36w + 162$
	Answer: **5. (*b*)** $2(w - 9)(w - 9)$

Factoring the Trinomial $ax^2 + bx + c$ with $a \neq 1$

The *ac* Method

Study the product at right.

$$(2x + 3)(3x + 4)$$

$$2x(3x + 4) + 3(3x + 4)$$

$$6x^2 + \mathbf{8}x + \mathbf{9}x + 12$$

$$\mathbf{6}x^2 + \mathbf{17}x + \mathbf{12}$$

Notice $\mathbf{6} \bullet \mathbf{12} = \mathbf{72} = \mathbf{9} \bullet \mathbf{8}$

To factor the trinomial $6x^2 + 17x + 12$ into the product of two binomials, without already knowing the binomials, we need to get "clues" from the 3 terms.

If we find two numbers whose product is **72** and whose sum is **17**, we'll be able to rewrite the trinomial as the polynomial with 4 terms the we see above and then factor by grouping or box method.

Product 72	
1	72
2	36
3	24
4	18
6	12
8	**9**
Sum 17	

This gives us that

$$\begin{aligned} 6x^2 + \mathbf{17}x + 12 &= 6x^2 + \mathbf{8}x + \mathbf{9}x + 12 \\ &= 2x(3x + 4) + 3(3x + 4) \\ &= (2x + 3)(3x + 4) \end{aligned}$$

or

	$3x$	4
$2x$	$6x^2$	$\mathbf{8}x$
3	$\mathbf{9}x$	12

In general, to factor the trinomial $ax^2 + bx + c$,

Step 1: Find two numbers that have a product of ac and a sum of b.
Step 2: Replace bx by the sum of the two terms with **coefficients** that are the two numbers found in Step 1.
Step 3: Factor the resulting four-term polynomial by grouping or by the box method.

Demonstration Problems

Factor the following trinomials.

6. (*a*) $6a^2 + 7a + 2$

$= 6a^2 + \quad + \quad + 2$

Product $6 \cdot 2 = 12$	
1	
2	
Sum 7	

7. (*a*) $15z^2 - 17z - 4$

$= 15z^2 + \quad + \quad - 4$

Product $15 \cdot {}^-4 = {}^-60$	
-1	
-2	
Sum $^-17$	

8. (*a*) $10t^2 + 13t - 4$

$= 10t^2 + \quad + \quad - 4$

Product $10 \cdot {}^-4 = {}^-40$	
-1	
-2	
Sum 13	

Practice Problems

Factor the following trinomials.

6. (*b*) $5x^2 + 17x + 6$

7. (*b*) $12b^2 + b - 1$

8. (*b*) $3r^2 + 2r + 1$

Answers: **6. (*b*)** $(x + 3)(5x + 2)$; **7. (*b*)** $(3b + 1)(4b - 1)$; **8. (*b*)** prime

Special Trinomials

Since $(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$, and
$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$,

we consider trinomials of the following form to be to be *perfect square trinomials*:

$$a^2 + 2ab + b^2 \qquad \text{and} \qquad a^2 - 2ab + b^2$$

First term is a square — Second term is twice the product of the bases of the first and last terms — Last term is a square

First term is a square — Second term is twice the product of the bases of the first and last terms — Last term is a square

If we can verify that a trinomial is a perfect square at a glance, then we can skip the lengthy techniques for factoring trinomials and write the factors immediately.

Example (*a*) Factor $4x^2 + 12x + 9$

$$4x^2 + 12x + 9$$

$$(2x)^2 \qquad 2 \bullet 2x \bullet 3 \qquad 3^2$$

Thus,

$$4x^2 + 12x + 9 = (2x + 3)^2$$

Example (*b*) Factor $x^2 - 6xy + 9y^2$

$$x^2 - 6xy + 9y^2$$

$$x^2 \qquad 2 \bullet x \bullet 3y \qquad (3y)^2$$

Thus,

$$x^2 - 6xy + 9y^2 = (x - 3y)^2$$

Demonstration Problems	*Practice Problems*
Factor the following trinomials. **9. (*a*)** $16a^2 + 8a + 1$ **10. (*a*)** $36y^2 - 12yz + z^2$	Factor the following trinomials. **9. (*b*)** $25x^2 + 30x + 9$ **10. (*b*)** $9a^2 - 6ab + b^2$
	Answers: **9. (*b*)** $(5x + 3)^2$; **10. (*b*)** $(3a - b)^2$

5.4 Factoring Binomials

Difference of Two Squares

Complete the table and notice the pattern that emerges.

	F	+	O	+	I	+	L	
$(x+1)(x-1) =$	x^2	+	$-1 \bullet x$	+	$1 \bullet x$	+	$-1 \bullet 1$	$= x^2 - 1$
$(2a-3)(2a+3) =$		+		+		+		
$(4m+5n)(4m-5n) =$		+		+		+		

Recall that the result of a subtraction problem is called a *difference*, so then $a^2 - b^2$ is a *difference of squares*.

$$a^2 - b^2 = (a+b)(a-b)$$

When we verify that a binomial is a difference of squares, we can use this pattern to write the factors immediately.

Example (*a*) Factor $x^2 - 9$.

$$\begin{aligned} &x^2 - 9 \\ &= x^2 - 3^2 \\ &= (x+3)(x-3) \end{aligned}$$

Example (*b*) Factor $4x^2 - 25$.

$$\begin{aligned} &4x^2 - 25 \\ &= (2x)^2 - 5^2 \\ &= (2x+5)(2x-5) \end{aligned}$$

Example (*c*) Factor $x^4 - 9y^2$.

$$\begin{aligned} &x^4 - 9y^2 \\ &= (x^2)^2 - (3y)^2 \\ &= (x^2+3y)(x^2-3y) \end{aligned}$$

Demonstration Problems	*Practice Problems*
Factor each polynomial.	Factor each polynomial.
1. (*a*) $a^2 - 4$	**1. (*b*)** $x^2 - 49$
2. (*a*) $y^2 - 9x^2$	**2. (*b*)** $16x^2 - y^2$
3. (*a*) $144n^2 - 1$	**3. (*b*)** $1 - 25w^2$
4. (*a*) $9a^2 - 16b^2$	**4. (*b*)** $25a^2 - 81b^2$
5. (*a*) $9a^4 - 16b^2$	**5. (*b*)** $25a^4 - 81b^2$

Answers: **1. (*b*)** $(x + 7)(x - 7)$; **2. (*b*)** $(4x + y)(4x - y)$; **3. (*b*)** $(1 + 5w)(1 - 5w)$; **4. (*b*)** $(5a + 9b)(5a - 9b)$; **5. (*b*)** $(5a^2 + 9b)(5a^2 - 9b)$;

Sum of Cubes and Difference of Cubes

Multiply the following and notice the pattern that emerges.

$(x + 1)(x^2 - x + 1) =$	$x \cdot x^2 \ + \ 1 \cdot x^2 \ + \ x \cdot -x \ + \ 1 \cdot -x \ + \ x \cdot 1 \ + \ 1 \cdot 1$	$= x^3 + 1$
$(3a + 5)(9a^2 - 15a + 25) =$	$+ \quad + \quad + \quad + \quad +$	
$(x - 1)(x^2 + x + 1) =$	$x \cdot x^2 \ + \ -1 \cdot x^2 \ + \ x \cdot x \ + \ -1 \cdot x \ + \ x \cdot 1 \ + \ -1 \cdot 1$	$= x^3 - 1$
$(a - 4)(a^2 + 4a + 16) =$	$+ \quad + \quad + \quad + \quad +$	

Factoring Patterns	
Sum of Cubes	*Difference of Cubes*
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

When we verify the binomial is a sum or a difference of cubes can use these patterns to write the factors immediately.

Example (*d*) Factor $x^3 + 27$.

$$x^3 + 27$$
$$= x^3 + 3^3$$
$$= (x + 3)(x^2 - 3x + 3^2)$$
$$= (x + 3)(x^2 - 3x + 9)$$

Example (*e*) Factor $8x^3 - 125$.

$$8x^3 - 125$$
$$= (2x)^3 - 5^3$$
$$= (2x - 5)((2x)^2 + 2x \cdot 5 + 5^2)$$
$$= (2x - 5)(4x^2 + 10x + 25)$$

Demonstration Problems	*Practice Problems*
Factor each binomial. **6. (*a*)** $a^3 + 64$	Factor each binomial. **6. (*b*)** $x^3 + 8$
7. (*a*) $y^3 - 27x^3$	**7. (*b*)** $8x^3 - y^3$
8. (*a*) $27n^3 - 1$	**8. (*b*)** $1 - 125w^3$
Answers: **6. (*b*)** $(x + 2)(x^2 - 2x + 4)$; **7. (*b*)** $(2x - y)(4x^2 + 2xy + y^2)$; **8. (*b*)** $(1 - 5w)(1 + 5w + 25w^2)$	

Factoring Completely

Example (*f*) Factor $2x^3 - 54$.

$$2x^3 - 54$$
$$= 2(x^3 - 27)$$
$$= 2(x - 3)(x^2 + 3x + 9)$$

Example (*g*) Factor $x^6 - y^6$.

$$x^6 - y^6$$
$$= (x^3)^2 - (y^3)^2$$
$$= (x^3 + y^3)(x^3 - y^3)$$
$$= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$$

Demonstration Problems	*Practice Problems*
Factor each polynomial. **9. (*a*)** $a^4 - 16$ **10. (*a*)** $3y^6 - 192x^6$	Factor each polynomial. **9. (*b*)** $x^4 - 81$ **10. (*b*)** $2x^6 - 128y^6$
Answers: **9. (*b*)** $(x^2 + 9)(x + 3)(x - 3)$; **10. (*b*)** $2(x - 2y)(x^2 + 2xy + 4y^2)(x + 2y)(x^2 - 2xy + 4y^2)$	

5.5 Choosing a Factoring Method

<table>
<tr><th colspan="4">Factoring Methods for Polynomials of 2 – 4 Terms</th></tr>
<tr><th>Number of terms</th><th>Step 1</th><th>Identify the polynomial or polynomial factor</th><th>Step 2</th></tr>
<tr><td rowspan="4">2</td><td rowspan="4">Factor out any GCF</td><td>Difference of squares $\Rightarrow$</td><td>$a^2 - b^2 = (a + b)(a - b)$</td></tr>
<tr><td>Sum of cubes $\Rightarrow$</td><td>$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$</td></tr>
<tr><td>Difference of cubes $\Rightarrow$</td><td>$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$</td></tr>
<tr><td>None of the above $\Rightarrow$</td><td>Cannot be factored by methods learned in this chapter.</td></tr>
<tr><td rowspan="3">3</td><td rowspan="3">Factor out any GCF</td><td>$x^2 + bx + c \Rightarrow$</td><td>$= (x \quad)(x \quad)$
Product c
Sum b</td></tr>
<tr><td>$ax^2 + bx + c \Rightarrow$</td><td>$ax^2 + bx + c$
$= ax^2 + b_1x + b_2x + c$
Then factor by grouping or box method.
Product ac
b_1 b_2
Sum b</td></tr>
<tr><td>None of the above $\Rightarrow$</td><td>Cannot be factored by methods learned in this chapter.</td></tr>
<tr><td>4</td><td>Factor out any GCF</td><td></td><td>Try to factor by grouping or box method.</td></tr>
</table>

Demonstration Problems	*Practice Problems*
Factor each polynomial.	Factor each polynomial.
1. (*a*) $a^2 - 25$	**1. (*b*)** $x^2 - 16$
2. (*a*) $8x^2 + 26x + 15$	**2. (*b*)** $6x^2 + x - 2$
3. (*a*) $n^3 - 1$	**3. (*b*)** $1 - w^3$
4. (*a*) $2a^4 - 32b^4$	**4. (*b*)** $243x^4 - 3y^4$
5. (*a*) $4ab + a - 1 - 4b$	**5. (*b*)** $2x^3 + x^2 - 50x - 25$

Answers: **1. (*b*)** $(x + 4)(x - 4)$; **2. (*b*)** $(3x + 2)(2x - 1)$; **3. (*b*)** $(1 - w)(1 + w + w^2)$; **4. (*b*)** $3(9x^2 + y^2)(3x + y)(3x - y)$; **5. (*b*)** $(x + 5)(x - 5)(2x + 1)$

5.6 Solving Equations by Factoring

Solve the following equations for *x*.

$2x = 0$	$-5x = 0$
$\frac{2}{3}x = 0$	$ax = 0$

If any two numbers have a product of zero, then one or both of them must be what number?

The answer to the preceding question can be formalized as

Zero Product Property*
If $ab = 0$, then $a = 0$ or $b = 0$.

* This property can be extended to any finite number of factors.

And now consider the equation

$$(x - 1)(x + 2) = 0$$

Can you find a solution by inspection (guessing and checking)?

Can you find a second solution?

Since $(x - 1)(x + 2) = 0$ is a product equal to 0, we can apply the Zero Product Property to solve it.

By the Zero Product Property,
if $(x - 1)(x + 2) = 0$, then

$x - 1 = 0$ or $x + 2 = 0$
$x = 1$ or $x = -2$

Solution set: $\{1, -2\}$

Demonstration Problems	*Practice Problems*
Solve **1. (*a*)** $(x - 3)(x - 5) = 0$	Solve **1. (*b*)** $(x + 1)(x - 2) = 0$
2. (*a*) $(2x - 1)(3x - 4) = 0$	**2. (*b*)** $(3x - 5)(4x - 1) = 0$
3. (*a*) $x(x + 5) = 0$	**3. (*b*)** $x(x - 7) = 0$

Answers: **1. (*b*)** $\{-1, 2\}$; **2. (*b*)** $\{\frac{5}{3}, \frac{1}{4}\}$; **3. (*b*)** $\{0, 7\}$

A *quadratic equation* in standard form has the form

$$ax^2 + bx + c = 0 \quad (a, b, c \text{ are real numbers and } a \neq 0)$$

To solve a quadratic equation,

1. Rewrite the equation, if necessary, in standard form.
2. Factor the polynomial completely.
3. Use the zero product property to write simple linear equations.
4. Solve the linear equations.

Demonstration Problems	*Practice Problems*
Solve.	Solve.
4. (*a*) $x^2 + 9x + 18 = 0$	**4. (*b*)** $x^2 + 9x + 20 = 0$
5. (*a*) $x^2 - x = 0$	**5. (*b*)** $x^2 - 2x = 0$
6. (*a*) $3x^2 - 2x - 1 = 0$	**6. (*b*)** $4x^2 + 3x - 1 = 0$

Answers: **4. (*b*)** $\{-4, -5\}$; **5. (*b*)** $\{0, 2\}$; **6. (*b*)** $\{-1, \frac{1}{4}\}$

Demonstration Problems	*Practice Problems*
Solve **7. (*a*)** $x^2 = 6x$	Solve **7. (*b*)** $x^2 = -5x$
8. (*a*) $x^2 + x = 56$	**8. (*b*)** $x^2 + 2x = 8$
9. (*a*) $5x^2 - 20 = 0$	**9. (*b*)** $3x^2 - 3 = 0$

Answers: **7. (*b*)** $\{0, -5\}$; **8. (*b*)** $\{-4, 2\}$; **9. (*b*)** $\{1, -1\}$;

Demonstration Problems	*Practice Problems*
Solve **10. (*a*)** $(3x - 12)(x + 7)(x - 2) = 0$	Solve **10. (*b*)** $(2x + 4)(x - 5)(x + 1) = 0$
11. (*a*) $(x + 2)(x - 6) = 20$	**11. (*b*)** $(x + 3)(2x - 1) = 15$

Answers: **10. (*b*)** $\{-2, 5, -1\}$; **11. (*b*)** $\{-\frac{9}{2}, 2\}$