

2.6 Applications

Solutions

<p>1. What number must be added to the numerator and subtracted from the denominator of $\frac{10}{23}$ to obtain a fraction equivalent to $\frac{6}{5}$?</p> <p>Let x = the number.</p> $\frac{10+x}{23-x} = \frac{6}{5}$ $5(10+x) = 6(23-x)$ $50 + 5x = 138 - 6x$ $\begin{array}{r} +6x \\ \hline 50 + 11x = 138 \\ -50 \quad \quad -50 \\ \hline 11x = 88 \\ \frac{11x}{11} = \frac{88}{11} \\ x = 8 \end{array}$ <p>The number is 8.</p>	<p>2. The numerator of a fraction is 4 less than the fraction's denominator, and this fraction is equivalent to $\frac{3}{5}$. Find the original fraction.</p>												
<p>3. Kevin can do the newspaper delivery in his neighborhood in two hours, while it takes Dennis four hours to do it. How long would it take to deliver the newspapers if they work together?</p> <p>Let x = the number of hours working together.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th></th> <th>Rate</th> <th>Time</th> <th>Production</th> </tr> </thead> <tbody> <tr> <td>Kevin</td> <td>$\frac{1}{2}$</td> <td>x</td> <td>$\frac{1}{2}x$</td> </tr> <tr> <td>Dennis</td> <td>$\frac{1}{4}$</td> <td>x</td> <td>$\frac{1}{4}x$</td> </tr> </tbody> </table> $\frac{1}{2}x + \frac{1}{4}x = 1$ $\frac{4}{1} \cdot \left(\frac{1}{2}x + \frac{1}{4}x \right) = \frac{4}{1} \cdot 1$ $2x + x = 4$ $3x = 4$ $\frac{3x}{3} = \frac{4}{3}$ $x = \frac{4}{3} = 1\frac{1}{3} \text{ hours}$ $= 1 \text{ hr} + \frac{1}{3} \cdot 60 \text{ mins}$ $= 1 \text{ hour } 20 \text{ minutes}$		Rate	Time	Production	Kevin	$\frac{1}{2}$	x	$\frac{1}{2}x$	Dennis	$\frac{1}{4}$	x	$\frac{1}{4}x$	<p>4. It takes machine A 9 hours to split the logs carried by a wagon. A faster machine B can do the job in 7 hours. How long would it take to split a wagonload of logs if both machines work together?</p>
	Rate	Time	Production										
Kevin	$\frac{1}{2}$	x	$\frac{1}{2}x$										
Dennis	$\frac{1}{4}$	x	$\frac{1}{4}x$										
<p>Answers: 1. 8; 3. 1 hour 20 minutes</p>													

5. The inlet pipe of a hot tub can fill the tub in 6 minutes. When the drain is opened the water drains out in 9 minutes. If the drain was left open by mistake, how long would it take to fill the tub?

Let x = the number of minutes working together.

	Rate •	Time =	Production
Inlet	$\frac{1}{6}$	x	$\frac{1}{6}x$
Drain	$\frac{1}{9}$	x	$\frac{1}{9}x$

$$\frac{1}{6}x - \frac{1}{9}x = 1$$

$$\frac{18}{1} \cdot \left(\frac{1}{6}x - \frac{1}{9}x \right) = \frac{18}{1} \cdot 1$$

$$3x - 2x = 18$$

$$x = 18 \text{ minutes}$$

6. It takes three hours for a pipe to fill a tank of water if the tank does not leak. Unfortunately our tank has a crack that could cause it to empty in 5 hours. How long would it take to completely fill this leaky tank?

7. A boat can travel 17 miles downstream in the same time it takes it to travel 13 miles to go upstream the same river. If the boat's speed is 45 mph in still water, what is the speed of the current?

Let x = the speed of the current.

	Rate •	Time =	Production
Up-stream	$45 - x$	$\frac{13}{45 - x}$	13
Down-stream	$45 + x$	$\frac{17}{45 + x}$	17

$$\frac{17}{45 + x} = \frac{13}{45 - x}$$

$$17(45 - x) = 13(45 + x)$$

$$765 - 17x = 585 + 13x$$

$$\begin{array}{r} 765 \\ -585 \\ \hline 180 \end{array} = \begin{array}{r} 585 + 13x \\ -585 \\ \hline 30x \end{array}$$

$$\frac{180}{30} = \frac{30x}{30}$$

$$x = 6 \text{ mph}$$

8. A cargo boat travels to an upstream port and back (135 miles in each direction) in 24 hours. The speed of the current in the river is a constant 3 mph. Find the speed of the cargo boat in still water.

Answers: 5. 18 minutes; 7. 6 mph