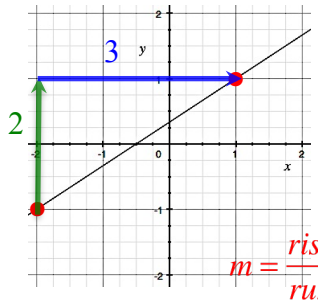
3.2 Geometric Characteristics of Lines

Solutions

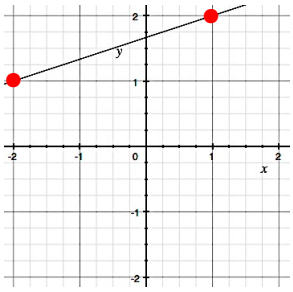
Determine the slope of each line.

1.

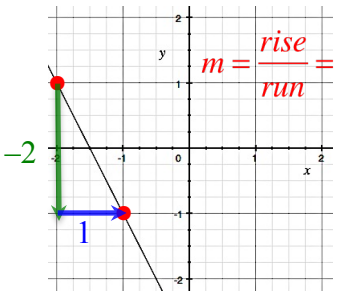


$$m = \frac{rise}{run} = \frac{2}{3}$$

2.

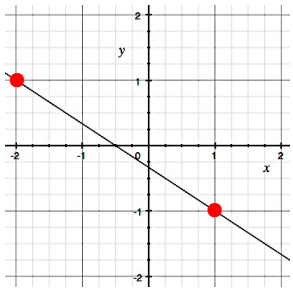


3.

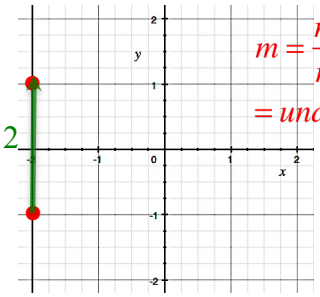


$$m = \frac{rise}{run} = \frac{-2}{1} = -2$$

4.

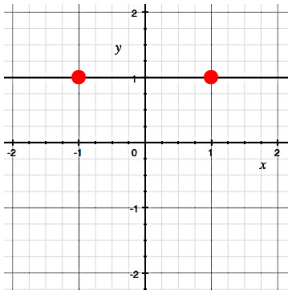


5.

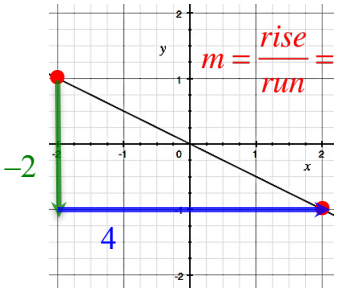


$$m = \frac{rise}{run} = \frac{2}{0}$$
$$= undefined$$

6.

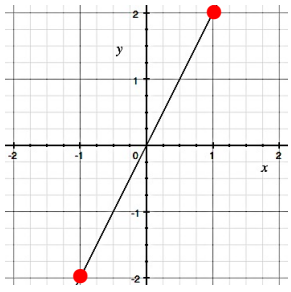


7.



$$m = \frac{rise}{run} = \frac{-2}{4} = -\frac{1}{2}$$

8.



Answers: **1.** $\frac{2}{3}$; **3.** –2; **5.** undefined; **7.** $-\frac{1}{2}$

Determine the slope of the line that passes through each pair of points.

9. (3, 4) and (5, 9)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-4}{5-3} = \frac{5}{2}$$

10. (0, 4) and (3, 8)

11. (2, –1) and (5, –1)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-1)}{5-2} = \frac{0}{3} = 0$$

12. (3, 4) and (3, –9)

Determine the slope and the y-intercept of each equation.

13. $y = \frac{2}{3}x + 5$ $\quad y = mx + b$

$m = \frac{2}{3}$ and $b = 5$

14. $y = -5x + 2$

15. $2x + 6y = 12$

$2x + 6y = 0x + 12$

$-2x \quad -2x$

$6y = -2x + 12$

$y = -\frac{2}{6}x + 2$

$m = -\frac{1}{3}$ and $b = 2$

16. $4x + 3y = 6$

17. $\frac{6y}{6} = \frac{12x}{6}$

$y = 2x + 0$

$m = 2$ and $b = 0$

18. $y = -8x$

19. $y = 15$

$y = 0x + 15$

$m = 0$ and $b = 15$

20. $y = -2$

Answers: **9.** $m = \frac{5}{2}$; **11.** $m = 0$; **13.** $m = \frac{2}{3}, b = 5$; **15.** $m = -\frac{1}{3}, b = 2$; **17.** $m = 2, b = 0$; **19.** $m = 0, b = 15$

Write an equation of the line with the given slope, m and y-intercept, b. Write it in slope-intercept form, $y = mx + b$.

21. $m = -5$ and $b = 10$ $\quad y = mx + b$

$y = -5x + 10$

22. $m = 3$ and $b = -1$

23. $m = \frac{5}{6}$ and $b = -2$ $\quad y = mx + b$

$y = \frac{5}{6}x - 2$

24. $m = -\frac{1}{4}$ and $b = 5$

25. $m = 0$ and $b = 8$ $\quad y = mx + b$

$y = 0x + 8$

$y = 8$

26. $m = 0$ and $b = -6$

27. $m = \frac{1}{7}$ and $b = 0$ $\quad y = mx + b$

$y = \frac{1}{7}x + 0$

$y = \frac{1}{7}x$

28. $m = -\frac{3}{4}$ and $b = 0$

Determine whether the following pairs of lines are parallel, perpendicular, or neither.

29. $y = -5x + 1$ and $y = \frac{1}{5}x + 12$

$m_\perp = -\frac{1}{m}$ $\quad m = -5,\ m_\perp = -\frac{1}{-5} = \frac{1}{5}$

Slopes are negative reciprocals to each other.

perpendicular

30. $y = \frac{1}{2}x + 5$ and $y = -2x - 2$

31. $y = -3x - 10$ and $y = 3x + 2$

$m_\perp = -\frac{1}{m}$ $\quad m = -3,\ m_\perp = -\frac{1}{-3} = \frac{1}{3}$

Slopes are not negative reciprocals to each other.

neither

32. $y = 4x - 5$ and $y = -4x - 4$

33. $y = 2x$ and $y = 2x + 12$

Slopes are the same.

parallel

34. $y = -3x - 4$ and $y = -3x$

35. $y = 15$ and $x = 12$

$y = 15$ is a horizontal line

$x = 12$ is a vertical line

perpendicular

36. $x = -5$ and $y = 1$

Answers: **21.** $y = -5x + 10$; **23.** $y = \frac{5}{6}x - 2$; **25.** $y = 8$; **27.** $y = \frac{1}{7}x$; **29.** perpendicular; **31.** neither; **31.** parallel; **35.** perpendicular

Write the equation of the line in slope-intercept form, $y = mx + b$ that has the following characteristics:

37. slope is -3 and passes through the point $(5, 6)$

$$y - y_1 = m(x - x_1)$$
$$y - 6 = -3(x - 5)$$
$$y - 6 = -3x + 15$$
$$+6 \qquad +6$$
$$y = -3x + 21$$

38. slope is 4 and passes through the point $(-1, 2)$

39. passes through the points $(-3, 1)$ and $(6, 4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{6 - (-3)} = \frac{3}{9} = \frac{1}{3}$$

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{1}{3}(x - (-3))$$
$$y - 1 = \frac{1}{3}x + \frac{1}{3} \cdot 3$$
$$y - 1 = \frac{1}{3}x + 1$$
$$+1 \qquad +1$$
$$y = \frac{1}{3}x + 2$$

40. passes through the points $(-1, 3)$ and $(2, -3)$

41. passes through $(2, 4)$ and is parallel to $y = \frac{1}{2}x - 1$.

$$m = \frac{1}{2}$$

$$y - y_1 = m(x - x_1)$$
$$y - 4 = \frac{1}{2}(x - 2)$$
$$y - 4 = \frac{1}{2}x - \frac{1}{2} \cdot 2$$
$$y - 4 = \frac{1}{2}x - 1$$
$$+4 \qquad +4$$
$$y = \frac{1}{2}x + 3$$

42. passes through $(2, 3)$ and is parallel to $y = 3x - 9$.

43. passes through $(1, 2)$ and is perpendicular to $y = \frac{1}{2}x - 1$.

$$m = -2$$
$$y - y_1 = m(x - x_1)$$
$$y - 2 = -2(x - 1)$$
$$y - 2 = -2x + 2$$
$$+2 \qquad +2$$
$$y = -2x + 4$$

44. passes through $(3, -2)$ and is perpendicular to $y = 3x - 9$.

Answers: **37.** $y = -3x + 21$; **39.** $y = \frac{1}{3}x + 2$; **41.** $y = \frac{1}{2}x + 3$; **43.** $y = -2x + 4$