

4.3 Operations with Radical Expressions

Solutions

Simplify the expressions.	
<p>1. $\sqrt{10} \cdot \sqrt{5} = \sqrt{10 \cdot 5} = \sqrt{50}$ $= \sqrt{5^2 \cdot 2} = 5\sqrt{2}$ or $= \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = \boxed{5\sqrt{2}}$</p>	<p>2. $\sqrt{35} \cdot \sqrt{5}$</p>
<p>3. $\sqrt[3]{2} \cdot \sqrt[3]{28} = \sqrt[3]{2 \cdot 28} = \sqrt[3]{56}$ $= \sqrt[3]{2^3 \cdot 7} = \sqrt[3]{2^3} \cdot \sqrt[3]{7} = \boxed{2\sqrt[3]{7}}$</p>	<p>4. $\sqrt[3]{4} \cdot \sqrt[3]{26}$</p>
<p>5. $\sqrt{5x^3} \cdot \sqrt{40x} = \sqrt{5x^3 \cdot 40x}$ $= \sqrt{200x^4} = \sqrt{100 \cdot 2(x^2)^2}$ $= 10x^2\sqrt{2}$ or $= \sqrt{2^3 5^2 (x^2)^2} = \sqrt{2^2 \cdot 2 \cdot 5^2 x^2 \cdot x^2}$ $= 2 \cdot 5x^2\sqrt{2} = \boxed{10x^2\sqrt{2}}$</p>	<p>6. $\sqrt{15x} \cdot \sqrt{3x^5}$</p>
<p>7. $\sqrt{2x^3y^3} \cdot \sqrt{54x^2y} = \sqrt{2x^3y^3 \cdot 54x^2y}$ $= \sqrt{108x^5y^4} = \sqrt{2^2 \cdot 3^3 x^5 y^4}$ $= \sqrt{2^2 \cdot 3^2 \cdot 2 \cdot (x^2)^2 \cdot x \cdot (y^2)^2}$ $= 2 \cdot 3x^2y^2 = 6x^2y^2$ or $= \sqrt{2^2 \cdot 3^2 \cdot 2 \cdot x^2 \cdot x^2 \cdot x \cdot y^2 \cdot y^2}$ $= \boxed{2 \cdot 3x^2y^2 = 6x^2y^2}$</p>	<p>8. $\sqrt{30x^5y^3} \cdot \sqrt{5x^3y^4}$</p>
<p>Answers: 1. $5\sqrt{2}$; 3. $2\sqrt[3]{7}$; 5. $10x^2\sqrt{2}$; 7. $6x^2y^2\sqrt{3x}$</p>	

Simplify the expressions.	
<p>9. $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = \sqrt{3^2} = \boxed{3}$</p>	<p>10. $\frac{\sqrt{50}}{\sqrt{2}}$</p>
<p>11. $\frac{\sqrt{3x^9}}{\sqrt{27x}} = \sqrt{\frac{3x^9}{27x}} = \sqrt{\frac{1\cancel{x} \cdot x^9}{9 \cdot 27 \cdot x}} = \sqrt{\frac{1 \cdot x^8}{9 \cdot 1}} = \sqrt{\frac{x^8}{9}} = \frac{\sqrt{x^8}}{\sqrt{9}} = \boxed{\frac{x^4}{3}}$</p> <p>Recall: $\frac{a^m}{a^n} = \frac{a^{m-n}}{1}$</p>	<p>12. $\frac{\sqrt{2x}}{\sqrt{32x^5}}$</p>
<p>13. $\frac{\sqrt[3]{16x^{14}}}{\sqrt[3]{2x^7}} = \sqrt[3]{\frac{16x^{14}}{2x^7}} = \sqrt[3]{8x^7} = \sqrt[3]{2^3 \cdot (x^2)^3 \cdot x} = \boxed{2x^2 \sqrt[3]{x}}$</p>	<p>14. $\frac{\sqrt[3]{54x^{25}}}{\sqrt[3]{2x^4}}$</p>
<p>15. $\frac{\sqrt{200x^8y^5z}}{\sqrt{10x^{10}y^2z}} = \sqrt{\frac{200x^8y^5z}{10x^{10}y^2z}} = \sqrt{\frac{200 \cdot x^8 \cdot y^5 \cdot z}{10 \cdot x^{10} \cdot y^2 \cdot z}} = \sqrt{\frac{20 \cdot 1 \cdot y^3 \cdot 1}{1 \cdot x^2 \cdot 1}} = \sqrt{\frac{20y^3}{x^2}} = \sqrt{\frac{2^2 \cdot 5 \cdot y^2 \cdot y}{x^2}} = \boxed{\frac{2y\sqrt{5y}}{x}}$</p>	<p>16. $\frac{\sqrt{300x^6y^7z^3}}{\sqrt{20x^4y^2z}}$</p>
<p>Answers: 9. 3; 10. $\frac{x^4}{3}$; 11. $\frac{x^4}{3}$; 12. $\frac{x^4}{3}$; 13. $2x^2 \sqrt[3]{x}$; 14. $\frac{2y\sqrt{5y}}{x}$</p>	

Simplify the expressions.

17. $4\sqrt{3} - 2\sqrt{3} = (4-2)\sqrt{3} = 2\sqrt{3}$

18. $5\sqrt{7} - \sqrt{7}$

19. $4\sqrt{x} + 2 + 6\sqrt{x} - 1 \cdot \sqrt{x}$
 $= (4+6-1)\sqrt{x} + 2$
 $= 9\sqrt{x} + 2$

20. $2\sqrt{y} + 5y - 4\sqrt{y} - 2\sqrt{y}$

21. $\sqrt{20} + \sqrt{45}$
 $= \sqrt{2^2 \cdot 5} + \sqrt{3^2 \cdot 5}$
 $= 2\sqrt{5} + 3\sqrt{5}$

22. $\sqrt{75} + \sqrt{108}$

23. $8\sqrt{45} + 7\sqrt{20} + 2\sqrt{5}$
 $= 8\sqrt{3^2 \cdot 5} + 7\sqrt{2^2 \cdot 5} + 2\sqrt{5}$
 $= 8 \cdot 3\sqrt{5} + 7 \cdot 2\sqrt{5} + 2\sqrt{5}$
 $= 24\sqrt{5} + 14\sqrt{5} + 2\sqrt{5}$
 $= (24+14+2)\sqrt{5}$
 $= 40\sqrt{5}$

24. $5\sqrt{12} - 2\sqrt{27} - 3\sqrt{3}$

Answers: 17. $2\sqrt{3}$; 19. $9\sqrt{x} + 2$; 21. $5\sqrt{5}$; 23. $40\sqrt{5}$

Simplify the expressions.

$$\begin{aligned}
 25. \quad & \sqrt{5}(x+\sqrt{5}) = \sqrt{5} \cdot x + \sqrt{5} \cdot \sqrt{5} \\
 & = x\sqrt{5} + \sqrt{5^2} = x\sqrt{5} + 5
 \end{aligned}$$

$$26. \quad \sqrt{2}(\sqrt{3} + \sqrt{2})$$

$$\begin{aligned}
 27. \quad & (2-2\sqrt{2})(1+\sqrt{3}) \\
 & = 2 \cdot 1 + 2 \cdot \sqrt{3} - 2\sqrt{2} \cdot 1 - 2\sqrt{2} \cdot \sqrt{3} \\
 & = 2 + 2\sqrt{3} - 2\sqrt{2} - 2\sqrt{6}
 \end{aligned}$$

$$28. \quad (5+\sqrt{2})(5-\sqrt{5})$$

$$\begin{aligned}
 29. \quad & (2+\sqrt{3})(2-\sqrt{3}) \\
 & = 2 \cdot 2 - 2\sqrt{3} + 2\sqrt{3} - \sqrt{3} \cdot \sqrt{3} \\
 & = 4 + 0 - \sqrt{3^2} \\
 & = 4 - 3 \\
 & = 1
 \end{aligned}$$

$$30. \quad (1+\sqrt{2})(1-\sqrt{2})$$

$$\begin{aligned}
 31. \quad & (2+\sqrt{3})^2 = (2+\sqrt{3})(2+\sqrt{3}) \\
 & = 2 \cdot 2 + 2\sqrt{3} + 2\sqrt{3} + \sqrt{3} \cdot \sqrt{3} \\
 & = 4 + 4\sqrt{3} + \sqrt{3^2} \\
 & = 4 + 4\sqrt{3} + 3 \\
 & = 7 + 4\sqrt{3}
 \end{aligned}$$

$$32. \quad (1+\sqrt{2})^2$$

Answers: 25. $x\sqrt{5} + 5$; 27. $2 + 2\sqrt{3} - 2\sqrt{2} - 2\sqrt{6}$; 29. 1; 31. $7 + 4\sqrt{3}$

Rationalize the denominators

$$33. \quad \frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5}}{\sqrt{5^2}} = \frac{4\sqrt{5}}{5}$$

$$34. \quad \frac{3}{\sqrt{2}}$$

$$35. \quad \frac{5}{\sqrt[3]{4}} = \frac{5}{\sqrt[3]{2^2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{5\sqrt[3]{2}}{\sqrt[3]{2^3}}$$

$$= \frac{5\sqrt[3]{2}}{2}$$

$$36. \quad \frac{1}{\sqrt[3]{9}}$$

$$37. \quad \frac{2}{5-\sqrt{3}} \cdot \frac{5+\sqrt{3}}{5+\sqrt{3}} = \frac{2(5+\sqrt{3})}{(5-\sqrt{3})(5+\sqrt{3})}$$

$$= \frac{2(5+\sqrt{3})}{25+5\sqrt{3}-5\sqrt{3}-\sqrt{3}\cdot\sqrt{3}} = \frac{2(5+\sqrt{3})}{25-\sqrt{3^2}}$$

$$= \frac{2(5+\sqrt{3})}{25-3} = \frac{2(5+\sqrt{3})}{22} = \frac{5+\sqrt{3}}{11}$$

$$38. \quad \frac{4}{1-\sqrt{2}}$$

$$39. \quad \frac{\sqrt{x}}{10+\sqrt{x}} \cdot \frac{10-\sqrt{x}}{10-\sqrt{x}} = \frac{\sqrt{x}(10-\sqrt{x})}{(10+\sqrt{x})(10-\sqrt{x})}$$

$$= \frac{10\sqrt{x}-\sqrt{x^2}}{100-10\sqrt{x}+10\sqrt{x}-\sqrt{x^2}} = \frac{10\sqrt{x}-x}{100-x}$$

$$40. \quad \frac{\sqrt{2}}{4-\sqrt{2}}$$

Answers: 33. $\frac{4\sqrt{5}}{5}$; 35. $\frac{5\sqrt[3]{2}}{2}$; 37. $\frac{5+\sqrt{3}}{11}$; 38. $\frac{10\sqrt{x}-x}{100-x}$