

4.4 Rational Exponents

Solutions

Write the following expressions using rational exponents only. Use negative exponents in #3, 4.	
1. $\sqrt[2]{7} = \boxed{7^{\frac{1}{2}}}$	2. $\sqrt[3]{5}$
Recall: $\sqrt[m]{a^n} = a^{\frac{n}{m}}$	
3. $\frac{1}{\sqrt[2]{5}} = \frac{1}{5^{\frac{1}{2}}} = \boxed{5^{-\frac{1}{2}}}$	4. $\frac{1}{\sqrt[3]{7}}$
Recall: $a^{-n} = \frac{1}{a^n}$	
Write the following expressions using radical notation only. Simplify where possible.	
5. $25^{\frac{1}{2}} = \sqrt[2]{25} = \sqrt{5^2} = \boxed{5}$	6. $8^{\frac{1}{3}}$
Recall: $\sqrt[m]{a^n} = a^{\frac{n}{m}}$	
7. $4y^{\frac{2}{3}} = \boxed{4\sqrt[3]{y^2}}$	8. $10x^{\frac{1}{5}}$
Answers: 1. $7^{\frac{1}{2}}$; 3. $5^{-\frac{1}{2}}$; 5. 5; 7. $4\sqrt[3]{y^2}$	

Simplify the expressions.

9. $\left(\frac{1}{9}\right)^{\frac{1}{2}} = \frac{1^{\frac{1}{2}}}{9^{\frac{1}{2}}} = \frac{1}{(3^2)^{\frac{1}{2}}} = \frac{1}{3^{2 \cdot \frac{1}{2}}} = \frac{1}{3}$

Recall: $(a^m)^n = a^{m \cdot n}$

10. $\left(\frac{1}{8}\right)^{\frac{1}{3}}$

11. $4^{\frac{3}{2}} = (2^2)^{\frac{3}{2}} = 2^{2 \cdot \frac{3}{2}} = 2^3 = 8$

12. $8^{\frac{2}{3}}$

13. $(x^4)^{\frac{1}{2}} = x^{4 \cdot \frac{1}{2}} = x^2$

14. $(x^9)^{\frac{1}{3}}$

15. $(a^6)^{\frac{2}{3}} = a^{6 \cdot \frac{2}{3}} = a^4$

16. $(m^8)^{\frac{3}{4}}$

Answers: 9. $\frac{1}{3}$; 11. 8; 13. x^2 ; 15. a^4

Simplify the expressions.

17. $x^{\frac{1}{2}} \cdot x^{\frac{3}{2}} = x^{\frac{1+3}{2}} = x^{\frac{4}{2}} = x^2$

18. $a^{\frac{1}{4}} \cdot a^{\frac{3}{4}}$

Recall: $a^m \cdot a^n = a^{m+n}$

19. $y^{\frac{7}{5}} \cdot y^{-\frac{1}{2}} = y^{\frac{7}{5} - \frac{1}{2}} = y^{\frac{7 \cdot 2 - 1 \cdot 5}{5 \cdot 2}} = y^{\frac{14 - 5}{10}} = y^{\frac{9}{10}}$

20. $3^{-\frac{3}{4}} \cdot 3^{\frac{1}{2}}$

21. $\frac{x^3}{x^{\frac{1}{2}}} = x^{3 - \frac{1}{2}} = x^{\frac{3 \cdot 2 - 1}{2}} = x^{\frac{6 - 1}{2}} = x^{\frac{5}{2}}$

22. $\frac{a^4}{a^{\frac{1}{3}}}$

Recall: $\frac{a^m}{a^n} = a^{m-n}$

23. $\sqrt[4]{x^2} = x^{\frac{2}{4}} = x^{\frac{1}{2}} = \sqrt{x} = \sqrt{x}$

24. $\sqrt[12]{x^4 y^4}$

Answers: 17. x^2 ; 19. $y^{\frac{9}{10}}$; 21. $x^{\frac{5}{2}}$; 23. \sqrt{x}

Write as a single radical.

$$\begin{aligned}
 25. \quad & \sqrt[3]{2} \cdot \sqrt{2} = 2^{\frac{1}{3}} \cdot 2^{\frac{1}{2}} = 2^{\frac{1}{3} + \frac{1}{2}} \\
 & = 2^{\frac{1 \cdot 2 + 1 \cdot 3}{3 \cdot 2} + \frac{1 \cdot 3}{2 \cdot 3}} = 2^{\frac{2}{6} + \frac{3}{6}} = 2^{\frac{5}{6}} \\
 & = \sqrt[6]{2^5} = \boxed{\sqrt[6]{32}}
 \end{aligned}$$

$$26. \quad \sqrt[3]{x} \cdot \sqrt{x}$$

$$\begin{aligned}
 27. \quad & \sqrt[3]{b^2} \cdot \sqrt[4]{b} = b^{\frac{2}{3}} \cdot b^{\frac{1}{4}} = b^{\frac{2}{3} + \frac{1}{4}} \\
 & = b^{\frac{2 \cdot 4 + 1 \cdot 3}{3 \cdot 4} + \frac{8}{12} + \frac{3}{12}} = b^{\frac{11}{12}} \\
 & = b^{\frac{11}{12}} = \boxed{\sqrt[12]{b^{11}}}
 \end{aligned}$$

$$28. \quad \sqrt[4]{a^2} \cdot \sqrt[5]{a}$$

$$\begin{aligned}
 29. \quad & \frac{\sqrt{x}}{\sqrt[3]{x}} = \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} = x^{\frac{1}{2} - \frac{1}{3}} = x^{\frac{1 \cdot 3 - 1 \cdot 2}{3 \cdot 2}} \\
 & = x^{\frac{3 - 2}{6}} = x^{\frac{1}{6}} = \boxed{\sqrt[6]{x}}
 \end{aligned}$$

$$30. \quad \frac{\sqrt[3]{x}}{\sqrt[5]{x}}$$

$$\begin{aligned}
 31. \quad & \sqrt[3]{\sqrt{x}} = (\sqrt{x})^{\frac{1}{3}} = \left(x^{\frac{1}{2}}\right)^{\frac{1}{3}} \\
 & = x^{\frac{1}{2} \cdot \frac{1}{3}} = x^{\frac{1}{6}} = \boxed{\sqrt[6]{x}}
 \end{aligned}$$

$$32. \quad \sqrt{\sqrt[3]{a^4}}$$

Answers: 25. $\sqrt[6]{32}$; 27. $\sqrt[12]{b^{11}}$; 29. $\sqrt[6]{x}$; 31. $\sqrt[6]{x}$