

4.6 Complex Numbers

Solutions

Write each expression as a complex number in the form, $a + bi$.

1.
$$\begin{aligned} -3 + \sqrt{-4} &= -3 + \sqrt{-1 \cdot 4} \\ &= -3 + \sqrt{-1} \cdot \sqrt{4} \\ &= -3 + i \cdot 2 \\ &= -3 + 2i \end{aligned}$$

2. $5 + \sqrt{-9}$

3.
$$\begin{aligned} 10 - \sqrt{-10} &= 10 - \sqrt{-1 \cdot 10} \\ &= 10 - \sqrt{-1} \cdot \sqrt{10} \\ &= 10 - i \cdot \sqrt{10} \end{aligned}$$

4. $3 - \sqrt{-7}$

5.
$$\begin{aligned} 2 + \sqrt{-45} &= 2 + \sqrt{-1 \cdot 3^2 \cdot 5} \\ &= 2 + \sqrt{-1} \cdot \sqrt{3^2} \cdot \sqrt{5} \\ &= 2 + i \cdot 3 \cdot \sqrt{5} \\ &= 2 + 3i\sqrt{5} \end{aligned}$$

6. $-2 - \sqrt{-27}$

7.
$$\begin{aligned} \sqrt{-80} &= \sqrt{-1 \cdot 2^4 \cdot 5} \\ &= \sqrt{-1} \cdot \sqrt{2^4} \cdot \sqrt{5} \\ &= i \cdot 2^2 \cdot \sqrt{5} \\ &= 4i\sqrt{5} \end{aligned}$$

8. $\sqrt{-200}$

Answers: 1. $-3 + 2i$; 3. $10 - i\sqrt{10}$; 5. $2 + 3i\sqrt{5}$; 7. $4i\sqrt{5}$

Simplify each expression. Write your answer in the form, $a + bi$.	
<p>9. $(4 + 5i) + (3 + 2i)$</p> $= 4 + 3 + 5i + 2i$ $= 7 + 7i$	<p>10. $(7 + 2i) + (4 + 3i)$</p>
<p>11. $(6 + 3i) - (2 + 4i)$</p> $= (6 + 3i) + -(2 + 4i)$ $= 6 + 3i + -2 + -4i$ $= 6 + -2 + 3i + -4i$ $= 4 - i$	<p>12. $(5 - 5i) - (3 + i)$</p>
<p>13. $(5 - 6i)(2 + i)$</p> $= 10 + 5i - 12i - 6i^2$ $= 10 - 7i - 6(-1)$ $= 10 - 7i + 6$ $= 16 - 7i$	<p>14. $(2 - 3i)(3 - 4i)$</p>
<p>15. $(5 - 6i)(5 + 6i)$</p> $= 25 + 30i - 30i - 36i^2$ $= 25 - 36(-1)$ $= 25 + 36$ $= 61$	<p>16. $(4 + 3i)(4 - 3i)$</p>
<p>17. $(1 - 2i)^2$</p> $= (1 - 2i)(1 - 2i)$ $= 1 - 2i - 2i + 4i^2$ $= 1 - 4i + 4(-1)$ $= 1 - 4i - 4$ $= -3 - 4i$	<p>18. $(3 + i)^2$</p>
Answers: 9. $7 + 7i$; 11. $4 - i$; 13. $16 - 7i$; 15. 61 ; 17. $-3 - 4i$	

Simplify each expression. Write your answer in the form, $a + bi$.

$$\begin{aligned}
 19. \quad \frac{3}{2i} \cdot \frac{i}{i} &= \frac{3i}{2i^2} = \frac{3i}{2(-1)} \\
 &= \frac{3i}{-2} = \boxed{-\frac{3}{2}i}
 \end{aligned}$$

$$20. \quad -\frac{5}{3i}$$

$$\begin{aligned}
 21. \quad \frac{6}{2+i} \cdot \frac{2-i}{2-i} &= \frac{6(2-i)}{(2+i)(2-i)} \\
 &= \frac{12-6i}{4-2i+2i-i^2} = \frac{12-6i}{4-(-1)} \\
 &= \frac{12-6i}{4+1} = \frac{12-6i}{5} = \boxed{\frac{12}{5} - \frac{6}{5}i}
 \end{aligned}$$

$$22. \quad \frac{9}{2-2i}$$

$$\begin{aligned}
 23. \quad \frac{5-i}{4-3i} \cdot \frac{4+3i}{4+3i} &= \frac{(5-i)(4+3i)}{(4-3i)(4+3i)} \\
 &= \frac{20+15i-4i-3i^2}{16+12i-12i-9i^2} = \frac{20+11i-3(-1)}{16-9(-1)} \\
 &= \frac{20+11i+3}{16+9} = \frac{23+11i}{25} = \boxed{\frac{23}{25} + \frac{11}{25}i}
 \end{aligned}$$

$$24. \quad \frac{4+2i}{1-5i}$$

$$\begin{aligned}
 25. \quad \frac{1+i}{1-i} \cdot \frac{1+i}{1+i} &= \frac{(1+i)(1+i)}{(1-i)(1+i)} \\
 &= \frac{1+i+i+i^2}{1+i-i-i^2} = \frac{1+2i+(-1)}{1-(-1)} \\
 &= \frac{2i}{1+1} = \frac{2i}{2} = \boxed{i}
 \end{aligned}$$

$$26. \quad \frac{3+i}{3-i}$$

Answers: 19. $-\frac{3}{2}i$; 21. $\frac{12}{5} - \frac{6}{5}i$; 23. $\frac{23}{25} + \frac{11}{25}i$; 25. i

Simplify.

27. $i^{12} = (i^4)^3 = 1^3 = 1$

28. i^{13}

29.
$$\begin{aligned} i^{50} &= i^{48} \cdot i^2 \\ &= (i^4)^{12} \cdot i^2 \\ &= 1^{12} \cdot (-1) \\ &= 1 \cdot (-1) \\ &= -1 \end{aligned}$$

30. i^{81}

31.
$$\begin{aligned} (3i)^4 &= 3^4 \cdot i^4 \\ &= 81 \cdot 1 \\ &= 81 \end{aligned}$$

32. $(2i)^5$

Answers: 27. 1; 29. -1; 31. 81