4.6 Complex Numbers Solutions

Write each expression as a complex number in the form, $a + bi$.

1.
$$-3 + \sqrt{-4} = -3 + \sqrt{-1 \cdot 4}$$
$$= -3 + \sqrt{-1} \cdot \sqrt{4}$$
$$= -3 + i \cdot 2$$
$$\boxed{= -3 + 2i}$$

2. $\quad 5 + \sqrt{-9}$

3.
$$10 - \sqrt{-10} = 10 - \sqrt{-1 \cdot 10}$$
$$= 10 - \sqrt{-1} \cdot \sqrt{10}$$
$$\boxed{= 10 - i \cdot \sqrt{10}}$$

4. $\quad 3 - \sqrt{-7}$

5.
$$2 + \sqrt{-45} = 2 + \sqrt{-1 \cdot 3^2 \cdot 5}$$
$$= 2 + \sqrt{-1} \cdot \sqrt{3^2} \cdot \sqrt{5}$$
$$= 2 + i \cdot 3 \cdot \sqrt{5}$$
$$\boxed{= 2 + 3i\sqrt{5}}$$

6. $\quad -2 - \sqrt{-27}$

7.
$$\sqrt{-80} = \sqrt{-1 \cdot 2^4 \cdot 5}$$
$$= \sqrt{-1} \cdot \sqrt{2^4} \cdot \sqrt{5}$$
$$= i \cdot 2^2 \cdot \sqrt{5}$$
$$\boxed{= 4i\sqrt{5}}$$

8. $\quad \sqrt{-200}$

Answers: **1.** $-3 + 2i$; **3.** $10 - i\sqrt{10}$; **5.** $2 + 3i\sqrt{5}$; **7.** $4i\sqrt{5}$

Simplify each expression. Write your answer in the form, $a + bi$.

9. $(4 + 5i) + (3 + 2i)$ $= 4 + 3 + 5i + 2i$ $\boxed{= 7 + 7i}$	**10.** $(7 + 2i) + (4 + 3i)$
11. $(6 + 3i) - (2 + 4i)$ $= (6 + 3i) + -(2 + 4i)$ $= 6 + 3i + -2 + -4i$ $= 6 + -2 + 3i + -4i$ $\boxed{= 4 - i}$	**12.** $(5 - 5i) - (3 + i)$
13. $(5 - 6i)(2 + i)$ $= 10 + 5i - 12i - 6i^2$ $= 10 - 7i - 6(-1)$ $= 10 - 7i + 6$ $\boxed{= 16 - 7i}$	**14.** $(2 - 3i)(3 - 4i)$
15. $(5 - 6i)(5 + 6i)$ $= 25 + 30i - 30i - 36i^2$ $= 25 - 36(-1)$ $= 25 + 36$ $\boxed{= 61}$	**16.** $(4 + 3i)(4 - 3i)$
17. $(1 - 2i)^2$ $= (1 - 2i)(1 - 2i)$ $= 1 - 2i - 2i + 4i^2$ $= 1 - 4i + 4(-1)$ $= 1 - 4i - 4$ $\boxed{= -3 - 4i}$	**18.** $(3 + i)^2$

Answers: **9**. $7 + 7i$; **11**. $4 - i$; **13**. $16 - 7i$; **15**. 61; **17**. $-3 - 4i$

Simplify each expression. Write your answer in the form, $a + bi$.

19.
$$\frac{3}{2i} \cdot \frac{i}{i} = \frac{3i}{2i^2} = \frac{3i}{2(-1)}$$
$$= \frac{3i}{-2} = \boxed{-\frac{3}{2}i}$$

20.
$$-\frac{5}{3i}$$

21.
$$\frac{6}{2+i} \cdot \frac{2-i}{2-i} = \frac{6(2-i)}{(2+i)(2-i)}$$
$$= \frac{12-6i}{4-2i+2i-i^2} = \frac{12-6i}{4-(-1)}$$
$$= \frac{12-6i}{4+1} = \frac{12-6i}{5} = \boxed{\frac{12}{5} - \frac{6}{5}i}$$

22.
$$\frac{9}{2-2i}$$

23.
$$\frac{5-i}{4-3i} \cdot \frac{4+3i}{4+3i} = \frac{(5-i)(4+3i)}{(4-3i)(4+3i)}$$
$$= \frac{20+15i-4i-3i^2}{16+12i-12i-9i^2} = \frac{20+11i-3(-1)}{16-9(-1)}$$
$$= \frac{20+11i+3}{16+9} = \frac{23+11i}{25} = \boxed{\frac{23}{25} + \frac{11}{25}i}$$

24.
$$\frac{4+2i}{1-5i}$$

25.
$$\frac{1+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{(1+i)(1+i)}{(1-i)(1+i)}$$
$$= \frac{1+i+i+i^2}{1+i-i-i^2} = \frac{1+2i+(-1)}{1-(-1)}$$
$$= \frac{2i}{1+1} = \frac{2i}{2} = \boxed{i}$$

26.
$$\frac{3+i}{3-i}$$

Answers: **19.** $-\frac{3}{2}i$; **21.** $\frac{12}{5} - \frac{6}{5}i$; **23.** $\frac{23}{25} + \frac{11}{25}i$; **25.** i

Simplify.

27. $i^{12} = (i^4)^3 = 1^3 = \boxed{1}$	**28.** i^{13}
29. $\begin{aligned} i^{50} &= i^{48} \cdot i^2 \\ &= (i^4)^{12} \cdot i^2 \\ &= 1^{12} \cdot (-1) \\ &= 1 \cdot (-1) \\ &= \boxed{-1} \end{aligned}$	**30.** i^{81}
31. $\begin{aligned} (3i)^4 &= 3^4 \cdot i^4 \\ &= 81 \cdot 1 \\ &= \boxed{81} \end{aligned}$	**32.** $(2i)^5$

Answers: **27**. 1; **29**. −1; **31**. 81