5.3 Solving Equations by Completing the Square

Solutions

Solve each equation by completing the square.

1. $x^2 + 6x = -5$

$x^2 + 2 \cdot 3x + 9 = -5 + 9$

$(x + 3)^2 = 4$

$\sqrt{(x+3)^2} = \pm\sqrt{4}$

$x + 3 = \pm 2$

$x + 3 = 2$ or $x + 3 = -2$

$\underline{-3}\ \underline{-3}$ $\underline{-3}\ \underline{-3}$

$x = -1$ or $x = -5$

$\{-1, -5\}$

2. $x^2 + 4x = 5$

3. $x^2 - 4x = 11$

$x^2 - 2 \cdot 2x + 4 = 11 + 4$

$(x - 2)^2 = 15$

$\sqrt{(x-2)^2} = \pm\sqrt{15}$

$x - 2 = \pm\sqrt{15}$

$\underline{+2}\ \underline{+2}$

$x = 2 \pm \sqrt{15}$

$\{2 \pm \sqrt{15}\}$

4. $x^2 - 10x = -15$

Answers: **1.** $\{-1, -5\}$; **3.** $\{2 \pm \sqrt{15}\}$

Solve each equation by completing the square.

5. $x^2 + 4x - 3 = 0$

$+3 \quad +3$

$x^2 + 4x = 3$

$x^2 + 2 \cdot 2x + 4 = 3 + 4$

$(x+2)^2 = 7$

$\sqrt{(x+2)^2} = \pm\sqrt{7}$

$x + 2 = \pm\sqrt{7}$

$-2 \quad -2$

$x = -2 \pm \sqrt{7}$

$\{-2 \pm \sqrt{7}\}$

6. $x^2 + 10x - 1 = 0$

7. $2x^2 - 11x + 5 = 0$

$\frac{2x^2 - 11x + 5}{2} = \frac{0}{2}$

$x^2 - \frac{11}{2}x + \frac{5}{2} = 0$

$-\frac{5}{2} \quad -\frac{5}{2}$

$x^2 - \frac{11}{2}x = -\frac{5}{2}$

$x^2 - 2 \cdot \frac{11}{4}x + \frac{121}{16} = -\frac{5}{2} + \frac{121}{16}$

$\left(x - \frac{11}{4}\right)^2 = \frac{81}{16}$

$\sqrt{\left(x - \frac{11}{4}\right)^2} = \pm\sqrt{\frac{81}{16}}$

$x - \frac{11}{4} = \pm\frac{9}{4}$

$+\frac{11}{4} \quad +\frac{11}{4}$

$x = \frac{11}{4} \pm \frac{9}{4}$

$\{5, \frac{1}{2}\}$

$x = \frac{20}{4} = 5$ or $x = \frac{2}{4} = \frac{1}{2}$

8. $2x^2 - 7x + 6 = 0$

Answers: **5.** $\{-2 \pm \sqrt{7}\}$; **7.** $\{\frac{1}{2}, 5\}$

Solve each equation by completing the square.

9. $2x^2 + 6x - 1 = 0$

$$\frac{2x^2+6x-1}{2} = \frac{0}{2}$$

$$x^2 + 3x - \frac{1}{2} = 0$$

$$\underline{+\frac{1}{2}} \quad \underline{+\frac{1}{2}}$$

$$x^2 + 3x = \frac{1}{2}$$

$$x^2 + 2 \cdot \frac{3}{2}x + \frac{9}{4} = \frac{1}{2} + \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{11}{4}$$

$$\sqrt{\left(x + \frac{3}{2}\right)^2} = \pm\sqrt{\frac{11}{4}}$$

$$x + \frac{3}{2} = \pm\frac{\sqrt{11}}{2}$$

$$\underline{-\frac{3}{2}} \quad \underline{-\frac{3}{2}}$$

$$x = \frac{-3 \pm \sqrt{11}}{2}$$

$\left\{\frac{-3 \pm \sqrt{11}}{2}\right\}$

10. $2x^2 - 5x + 1 = 0$

11. $3x^2 - 2x + 3 = 0$

$$\frac{3x^2-2x+3}{3} = \frac{0}{3}$$

$$x^2 - \frac{2}{3}x + 1 = 0$$

$$\underline{-1} \quad \underline{-1}$$

$$x^2 - \frac{2}{3}x = -1$$

$$x^2 - 2 \cdot \frac{1}{3}x + \frac{1}{9} = -1 + \frac{1}{9}$$

$$\left(x - \frac{1}{3}\right)^2 = -\frac{8}{9}$$

$$\sqrt{\left(x - \frac{1}{3}\right)^2} = \pm\sqrt{-\frac{8}{9}}$$

$$x - \frac{1}{3} = \pm\frac{2i\sqrt{2}}{3}$$

$$\underline{+\frac{1}{3}} \quad \underline{+\frac{1}{3}}$$

$$x = \frac{1 \pm 2i\sqrt{2}}{3} \text{ or } \frac{1}{3} \pm \frac{2\sqrt{2}}{3}i$$

$\left\{\frac{1}{3} \pm \frac{2\sqrt{2}}{3}i\right\}$

12. $4x^2 + 2x + 3 = 0$

Answers: **9.** $\left\{\frac{-3 \pm \sqrt{11}}{2}\right\}$; **11.** $\left\{\frac{1}{3} \pm \frac{2\sqrt{2}}{3}i\right\}$