

5.4 The Quadratic Formula

Solutions

Solve each equation by quadratic formula.

1. $1x^2 + 4x - 5 = 0$

$a = 1$ $b = 4$ $c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot (-5)}}{2 \cdot 1}$$

$$= \frac{-4 \pm \sqrt{16 + 20}}{2} = \frac{-4 \pm \sqrt{36}}{2}$$

$$= \frac{-4 \pm 6}{2}$$

$$x = \frac{-4 + 6}{2} \quad \text{or} \quad x = \frac{-4 - 6}{2}$$

$$x = \frac{2}{2} = 1 \quad \text{or} \quad x = \frac{-10}{2} = -5$$

$$\{1, -5\}$$

2. $x^2 - 4x + 3 = 0$

3. $1x^2 - 3x - 3 = 0$

$a = 1$ $b = -3$ $c = -3$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1}$$

$$= \frac{3 \pm \sqrt{9 + 12}}{2} = \frac{3 \pm \sqrt{21}}{2}$$

$$\left\{ \frac{3 \pm \sqrt{21}}{2} \right\}$$

4. $x^2 + 5x - 2 = 0$

Answers: 1. $\{1, -5\}$; 3. $\left\{ \frac{3 \pm \sqrt{21}}{2} \right\}$

Solve each equation by the quadratic formula.

5. $2x^2 + 3x = 2$

$$\underline{-2} \quad \underline{-2}$$

$$2x^2 + 3x - 2 = 0$$

$$a = 2 \quad b = 3 \quad c = -2$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot (-2)}}{2 \cdot 2}$$

$$= \frac{-3 \pm \sqrt{9 + 16}}{4} = \frac{-3 \pm \sqrt{25}}{4} = \frac{-3 \pm 5}{4}$$

$$x = \frac{-3 + 5}{4} \quad \text{or} \quad x = \frac{-3 - 5}{4}$$

$$x = \frac{2}{4} = \frac{1}{2} \quad \text{or} \quad x = \frac{-8}{4} = -2$$

$$\boxed{\left\{ \frac{1}{2}, -2 \right\}}$$

6. $3x^2 + 14x = 5$

7. $2x^2 = 2x + 2$

$$2x^2 \quad \underline{-2x} \quad \underline{-2x}$$

$$2x^2 - 2x - 2 = 0$$

$$a = 2 \quad b = -2 \quad c = -2$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 2 \cdot (-2)}}{2 \cdot 2}$$

$$= \frac{2 \pm \sqrt{4 + 16}}{4} = \frac{2 \pm \sqrt{20}}{4} = \frac{2 \pm 2\sqrt{5}}{4}$$

$$= \frac{2(1 \pm \sqrt{5})}{2 \cdot 2} = \frac{1 \pm \sqrt{5}}{2}$$

$$\boxed{\left\{ \frac{1 \pm \sqrt{5}}{2} \right\}}$$

8. $2x^2 + 4x = -1$

Answers: 5. $\{-2, \frac{1}{2}\}$; 7. $\{\frac{1 \pm \sqrt{5}}{2}\}$

Solve each equation by the quadratic formula.

9. $2x^2 + 1x + 2 = 0$

$$a = 2 \quad b = 1 \quad c = 2$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot (2)}}{2 \cdot 2}$$

$$= \frac{-1 \pm \sqrt{1 - 16}}{4} = \frac{-1 \pm \sqrt{-15}}{4} = \frac{-1 \pm i\sqrt{15}}{4}$$

$$x = -\frac{1}{4} \pm \frac{\sqrt{15}}{4}i$$

$$\left\{ -\frac{1}{4} \pm \frac{\sqrt{15}}{4}i \right\}$$

10. $3x^2 - 2x + 1 = 0$

11. $1x^2 + 2x + 2 = 0$

$$a = 1 \quad b = 2 \quad c = 2$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (2)}}{2 \cdot 1}$$

$$= \frac{-2 \pm \sqrt{4 - 8}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2}$$

$$x = \frac{-2}{2} \pm \frac{2i}{2} = -1 \pm i$$

$$\{-1 \pm i\}$$

12. $4x^2 + 2x + 1 = 0$

Answers: 9. $\left\{ -\frac{1}{4} \pm \frac{\sqrt{15}}{4}i \right\}$; 11. $\{-1 \pm i\}$

Determine the x -intercepts and the vertex. You do not have to graph the equations.

13. $y = x^2 - 2x$

x -intercepts:

$$0 = x^2 - 2x$$

$$0 = x(x - 2)$$

$$x = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = \frac{+2}{+2} = 2$$

$$\text{vertex: } x = \frac{0+2}{2} = \frac{2}{2} = 1$$

$$\begin{aligned} y &= x^2 - 2x = 1^2 - 2 \cdot 1 \\ &= 1 - 2 \\ &= -1 \end{aligned}$$

x -intercepts: $(0, 0)$ and $(2, 0)$

vertex: $(1, -1)$

14. $y = x^2 - 6x$

x -intercepts:

vertex:

15. $y = x^2 - 2x - 15$

x -intercepts:

$$0 = x^2 - 2x - 15$$

$$0 = (x + 3)(x - 5)$$

$$x + 3 = 0 \quad \text{or} \quad x - 5 = 0$$

$$x = \frac{-3}{-3} = -3 \quad \text{or} \quad x = \frac{+5}{+5} = 5$$

$$\text{vertex: } x = \frac{-3+5}{2} = \frac{2}{2} = 1$$

$$\begin{aligned} y &= x^2 - 2x - 15 = 1^2 - 2 \cdot 1 - 15 \\ &= 1 - 2 - 15 \\ &= -16 \end{aligned}$$

x -intercepts: $(-3, 0)$ and $(5, 0)$

vertex: $(1, -16)$

16. $y = x^2 + 6x + 8$

x -intercepts:

vertex:

Answer: **13.** $(0, 0), (2, 0)$, vertex: $(1, -1)$; **15.** $(-3, 0), (5, 0)$, vertex: $(1, -16)$

Determine the x -intercepts and the vertex. Write your answer to the nearest tenth. You do not have to graph the equations.

17. $y = x^2 - 4x - 2$

x -intercepts:

$$0 = x^2 - 4x - 2$$

$$0 = 1x^2 - 4x - 2$$

$$a = 1 \quad b = -4 \quad c = -2$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$= \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 2\sqrt{6}}{2}$$

$$= \frac{4}{2} \pm \frac{2\sqrt{6}}{2} = 2 \pm \sqrt{6}$$

$$x = 2 + \sqrt{6} \qquad x = 2 - \sqrt{6}$$

$$\approx 4.4 \qquad \approx -0.4$$

vertex:

$$x = \frac{2 + \sqrt{6} + 2 - \sqrt{6}}{2} = \frac{2 + 2}{2} = \frac{4}{2} = 2$$

$$y = x^2 - 4x - 2 = 2^2 - 4 \cdot 2 - 2$$

$$= 4 - 8 - 2$$

$$= -6$$

x -intercepts: $(4.4, 0)$ and $(-0.4, 0)$

vertex: $(2, -6)$

18. $y = x^2 - 2x - 2$

x -intercepts:

vertex:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Answer: 17. $(-0.4, 0)$, $(4.4, 0)$, vertex: $(2, -6)$