

## 5.5 Non-linear Inequalities

## Solutions

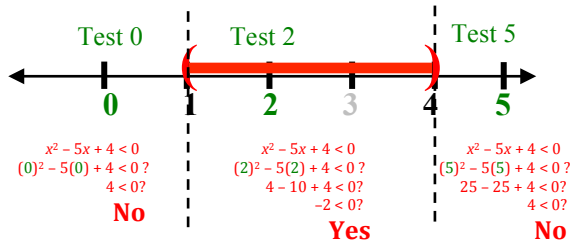
Solve and graph the solution set.

1.  $x^2 - 5x + 4 < 0$

$$(x - 1)(x - 4) = 0$$

$$x - 1 = 0 \quad \text{or} \quad x - 4 = 0$$

$$\begin{array}{r} +1 \quad +1 \\ x = 1 \end{array} \quad \text{or} \quad \begin{array}{r} +4 \quad +4 \\ x = 4 \end{array}$$



Interval notation:  $(1, 4)$

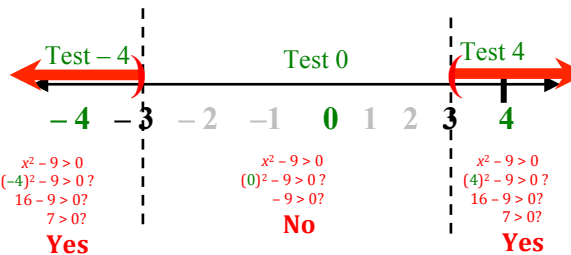
2.  $x^2 - 3x - 4 < 0$

3.  $x^2 - 9 > 0$

$$(x + 3)(x - 3) = 0$$

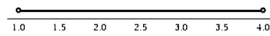
$$x + 3 = 0 \quad \text{or} \quad x - 3 = 0$$

$$\begin{array}{r} -3 \quad -3 \\ x = -3 \end{array} \quad \text{or} \quad \begin{array}{r} +3 \quad +3 \\ x = 3 \end{array}$$

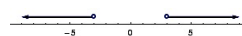


Interval notation:  $(-\infty, -3) \cup (3, \infty)$

Answers: 1.  $(1, 4)$



3.  $(-\infty, -3) \cup (3, \infty)$



Solve and graph the solution set.

5.  $x^2 - 2x \geq 3$

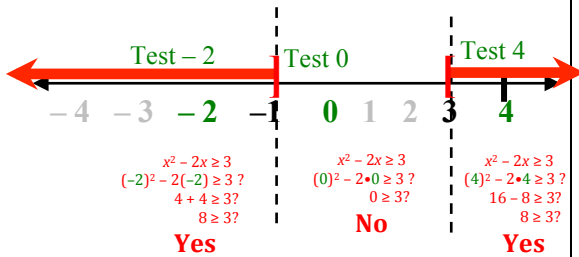
$\quad \quad \quad \underline{-3} \quad \underline{-3}$

$x^2 - 2x - 3 \geq 0$

$(x + 1)(x - 3) = 0$

$x + 1 = 0 \quad \text{or} \quad x - 3 = 0$

$\quad \quad \underline{-1} \quad \underline{-1} \quad \quad \quad \underline{+3} \quad \underline{+3}$   
 $x = -1 \quad \text{or} \quad x = 3$



Interval notation:  $(-\infty, -1] \cup [3, \infty)$

6.  $x^2 - 10x \geq 24$

7.  $x^2 + 9 < 0$

$1x^2 + 0x + 9 < 0$

$a = 1 \quad b = 0 \quad c = 9$

$x = \frac{-0 \pm \sqrt{0^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$

$= \frac{0 \pm \sqrt{0 - 36}}{2} = \frac{0 \pm \sqrt{-36}}{2} = \frac{\pm 6i}{2} = \pm 3i$

No real boundary points. Thus,

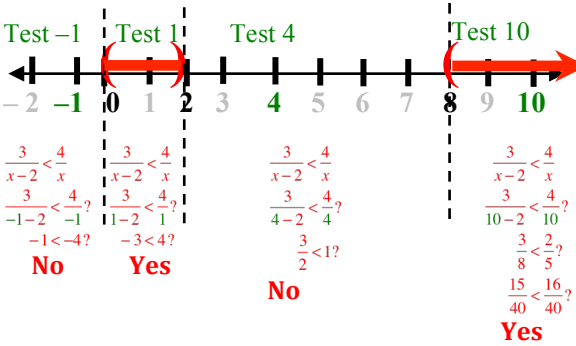
solution set:  $\emptyset$

8.  $x^2 + 4 < 0$

Answers: 5.  $(-\infty, -1] \cup [3, \infty)$  ; 7.  $\emptyset$

Solve and graph the solution set.

$$\begin{aligned}
 9. \quad & \frac{3}{x-2} < \frac{4}{x} \\
 & \frac{3}{x-2} = \frac{4}{x} \\
 & \quad \quad \frac{-4}{x} \quad \frac{-4}{x} \\
 & \frac{3}{x-2} - \frac{4}{x} = 0 \\
 & \frac{3}{x-2} \cdot \frac{x}{x} - \frac{4}{x} \cdot \frac{x-2}{x-2} = 0 \\
 & \frac{3x}{x(x-2)} - \frac{4(x-2)}{x(x-2)} = 0 \\
 & \frac{3x}{x(x-2)} - \frac{4x-8}{x(x-2)} = 0 \\
 & \frac{3x-4x+8}{x(x-2)} = 0 \\
 & \frac{-x+8}{x(x-2)} = 0 \\
 & -x+8 = 0 \quad \text{or} \quad x=0 \quad \text{or} \quad x-2 = 0 \\
 & \quad \quad \frac{-8}{-x} \quad \frac{-8}{-8} \quad \quad \quad \frac{+2}{+2} \quad \frac{+2}{+2} \\
 & -x = -8 \quad \text{or} \quad x=0 \quad \text{or} \quad x = 2 \\
 & -1(-x) = -1(-8) \\
 & \quad \quad \quad x = 8
 \end{aligned}$$



Interval notation:  $(0, 2) \cup (8, \infty)$

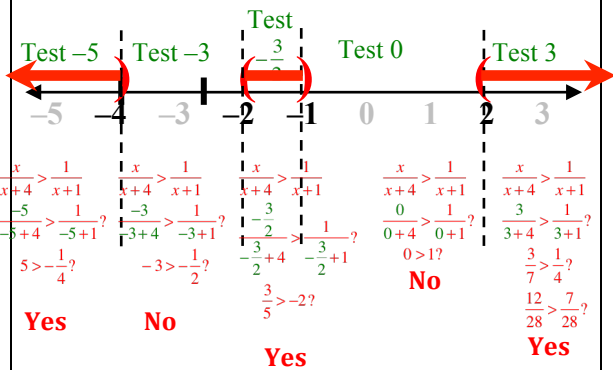
$$10. \quad \frac{1}{x+1} < \frac{2}{x}$$

Answer: 9.  $(0, 2) \cup (8, \infty)$

Solve and graph the solution set.

$$\begin{aligned}
 11. \quad \frac{x}{x+4} &> \frac{1}{x+1} \\
 \frac{x}{x+4} &= \frac{1}{x+1} \\
 \frac{x}{x+4} - \frac{1}{x+1} &= 0 \\
 \frac{x}{x+4} \cdot \frac{x+1}{x+1} - \frac{1}{x+1} \cdot \frac{x+4}{x+4} &= 0 \\
 \frac{x(x+1)}{(x+4)(x+1)} - \frac{x+4}{(x+4)(x+1)} &= 0 \\
 \frac{x^2+x-x-4}{(x+4)(x+1)} &= 0 \\
 \frac{x^2-4}{(x+4)(x+1)} &= 0 \\
 \frac{(x-2)(x+2)}{(x+4)(x+1)} &= 0
 \end{aligned}$$

$$\begin{aligned}
 x-2 &= 0 \text{ or } x+2 = 0 \text{ or } x+4 = 0 \text{ or } x+1 = 0 \\
 \frac{+2}{+2} \quad \frac{-2}{-2} \quad \frac{-4}{-4} \quad \frac{-1}{-1} \\
 x &= 2 \text{ or } x = -2 \text{ or } x = -4 \text{ or } x = -1
 \end{aligned}$$



$$(-\infty, -4) \cup (-2, -1) \cup (2, \infty)$$

$$12. \quad \frac{x}{x+9} > \frac{1}{x+1}$$

Answer: 11.  $(-\infty, -4) \cup (-2, -1) \cup (2, \infty)$