5.5 Non-linear Inequalities

Solutions

Solve and graph the solution set.

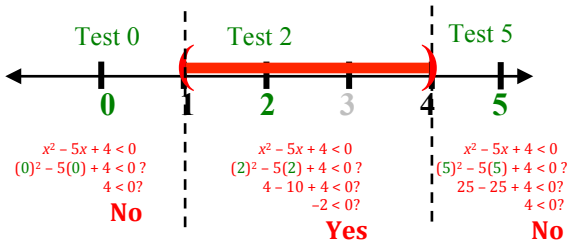
1. $x^2 - 5x + 4 < 0$

$(x - 1)(x - 4) = 0$

$x - 1 = 0$ or $x - 4 = 0$

$+1\ +1$ $\quad$ $+4\ +4$

$x = 1$ or $x = 4$



Interval notation: $(1, 4)$

2. $x^2 - 3x - 4 < 0$

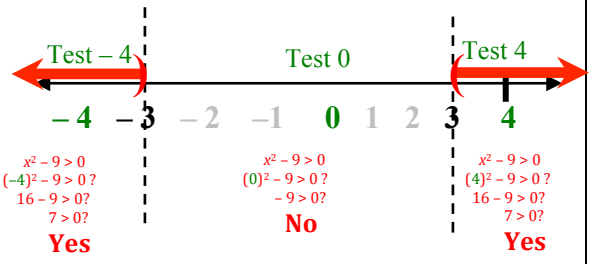
3. $x^2 - 9 > 0$

$(x + 3)(x - 3) = 0$

$x + 3 = 0$ or $x - 3 = 0$

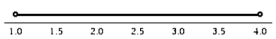
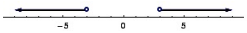
$-3\ -3$ $\quad$ $+3\ +3$

$x = -3$ or $x = 3$



Interval notation: $(-\infty, -3) \cup (3, \infty)$

4. $x^2 - 4 > 0$

Answers: **1.** $(1, 4)$  ; **3.** $(-\infty, -3) \cup (3, \infty)$ 

Solve and graph the solution set.

5. $x^2 - 2x \geq 3$

$-3 \quad -3$

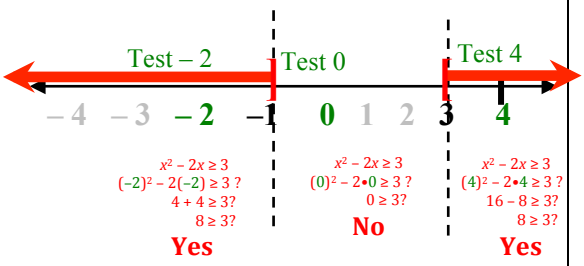
$x^2 - 2x - 3 \geq 0$

$(x + 1)(x - 3) = 0$

$x + 1 = 0$ or $x - 3 = 0$

$-1 \quad -1 \qquad +3 \quad +3$

$x = -1$ or $x = 3$



Interval notation: $(-\infty, -1] \cup [3, \infty)$

6. $x^2 - 10x \geq 24$

7. $x^2 + 9 < 0$

$1x^2 + 0x + 9 < 0$

$a = 1 \quad b = 0 \quad c = 9$

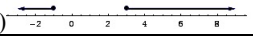
$$x = \frac{-0 \pm \sqrt{0^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$$

$$= \frac{0 \pm \sqrt{0 - 36}}{2} = \frac{0 \pm \sqrt{-36}}{2} = \frac{\pm 6i}{2} = \pm 3i$$

No real boundary points. Thus,

solution set: $\varnothing$

8. $x^2 + 4 < 0$

Answers: **5.** $(-\infty, -1] \cup [3, \infty)$  ; **7.** $\varnothing$

Solve and graph the solution set.

9. $\dfrac{3}{x-2} < \dfrac{4}{x}$

$\dfrac{3}{x-2} = \dfrac{4}{x}$

$-\dfrac{4}{x} \quad -\dfrac{4}{x}$

$\dfrac{3}{x-2} - \dfrac{4}{x} = 0$

$\dfrac{3}{x-2} \cdot \dfrac{x}{x} - \dfrac{4}{x} \cdot \dfrac{x-2}{x-2} = 0$

$\dfrac{3x}{x(x-2)} - \dfrac{4(x-2)}{x(x-2)} = 0$

$\dfrac{3x}{x(x-2)} - \dfrac{4x-8}{x(x-2)} = 0$

$\dfrac{3x-4x+8}{x(x-2)} = 0$

$\dfrac{-x+8}{x(x-2)} = 0$

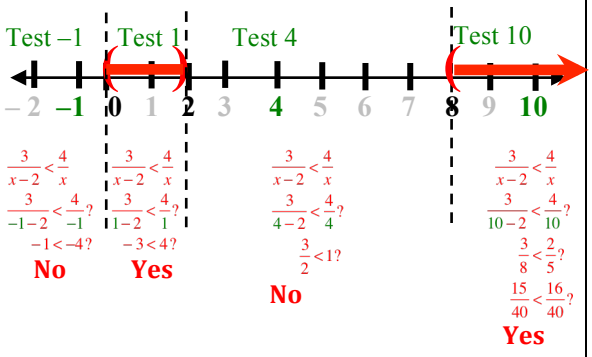
$-x + 8 = 0$ or $x = 0$ or $x - 2 = 0$

$-8 \quad -8 \qquad\qquad +2 \quad +2$

$-x = -8$ or $x = 0$ or $x = 2$

$-1(-x) = -1(-8)$

$x = 8$



Interval notation: $(0, 2) \cup (8, \infty)$

10. $\dfrac{1}{x+1} < \dfrac{2}{x}$

Answer: **9.** $(0, 2) \cup (8, \infty)$

Solve and graph the solution set.

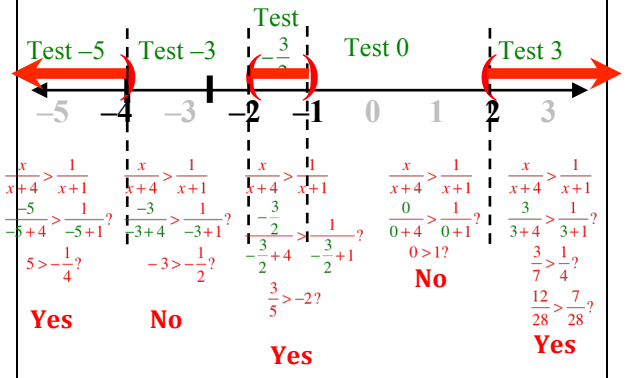
11. $\dfrac{x}{x+4} > \dfrac{1}{x+1}$

$$\frac{x}{x+4} = \frac{1}{x+1}$$
$$-\frac{1}{x+1} \qquad -\frac{1}{x+1}$$
$$\frac{x}{x+4} - \frac{1}{x+1} = 0$$
$$\frac{x}{x+4}\cdot\frac{x+1}{x+1} - \frac{1}{x+1}\cdot\frac{x+4}{x+4} = 0$$
$$\frac{x(x+1)}{(x+4)(x+1)} - \frac{x+4}{(x+4)(x+1)} = 0$$
$$\frac{x^2+x-x-4}{(x+4)(x+1)} = 0$$
$$\frac{x^2-4}{(x+4)(x+1)} = 0$$
$$\frac{(x-2)(x+2)}{(x+4)(x+1)} = 0$$

$x-2=0$ or $x+2=0$ or $x+4=0$ or $x+1=0$

$+2\ +2 \qquad -2\ -2 \qquad -4\ -4 \qquad -1\ -1$

$x = 2$ or $x = -2$ or $x = -4$ or $x = -1$



$$(-\infty, -4) \cup (-2, -1) \cup (2, \infty)$$

12. $\dfrac{x}{x+9} > \dfrac{1}{x+1}$

Answer: **11.** $(-\infty, -4) \cup (-2, -1) \cup (2, \infty)$