

5.6 Applications

Solutions

1. A whole number increased by its square is two more than twice itself. Find the number.

Let x = the whole number

$$\begin{aligned}
 x + x^2 &= 2x + 2 \\
 x^2 + x &= 2x + 2 \\
 \underline{-2x} \quad \underline{-2x} & \\
 x^2 - x &= 2 \\
 \underline{-2} \quad \underline{-2} & \\
 x^2 - x - 2 &= 0 \\
 (x + 1)(x - 2) &= 0 \\
 x + 1 = 0 &\text{ or } x - 2 = 0 \\
 \underline{-1} \quad \underline{-1} \quad \underline{+2} \quad \underline{+2} & \\
 x &= -1 \text{ or } x = 2
 \end{aligned}$$

↑
Not a
whole
number.

The whole number is 2.

2. Three less than the square of a whole number is equal to twice the number. Find the number.

Let x = the whole number

Answer: **1.** The whole number is 2.

3. If the area of a rectangle is 91 m^2 and its length is one meter shorter than twice its width, what are the dimensions of the rectangle?

Let $x = \text{width}$

then, $2x - 1 = \text{length}$

$$\text{Area} = \text{Length} \times \text{Width}$$

Product	
-182	
-1	182
-2	91
-7	26
-13	14
13	-14
Sum -1	

$$2x \quad 13$$

x	$2x^2$	$13x$
-7	$-14x$	91

$$\begin{aligned}
 91 &= (2x - 1)x \\
 91 &= 2x^2 - x \\
 -91 & \qquad \qquad -91 \\
 0 &= 2x^2 - x - 91 \\
 0 &= 2x^2 + 13x - 14x - 91 \\
 0 &= (2x + 13)(x - 7) \\
 2x + 13 &= 0 \quad \text{or} \quad x - 7 = 0 \\
 \underline{-13} \quad \underline{-13} & \qquad \underline{+7} \quad \underline{+7} \\
 2x &= -13 \quad x = 7 \\
 \frac{2x}{2} &= \frac{-13}{2} \\
 x &= \frac{-13}{2}
 \end{aligned}$$

↑
Width cannot be negative.

$$\begin{aligned}
 x &= 7 \\
 \Rightarrow 2x - 1 & \\
 &= 2 \cdot 7 - 1 \\
 &= 14 - 1 \\
 &= 13
 \end{aligned}$$

Length is 13m, width is 7m.

4. The area of a rectangle is 70 ft^2 and its length is one foot shorter than three times its width. What are the dimensions of the rectangle?

Let $x = \text{width}$

then, $\quad = \text{length}$

$$\text{Area} = \text{Length} \times \text{Width}$$

Answer: 3. Length is 13 m, width is 7 m.

5. Together, it takes Angel and Bart four hours to mow a certain lawn. It takes Angel 6 more hours than Bart to do the job alone. How long would each need to mow the lawn working alone?

Let x = time it takes Bart to mow alone
then

$x + 6$ = time it takes Angel to mow alone

	Rate	Time (WT)	Amount of work done
Bart	$\frac{1}{x}$	4	$\frac{4}{x}$
Angel	$\frac{1}{x+6}$	4	$\frac{4}{x+6}$
Total			$\frac{4}{x} + \frac{4}{x+6} = 1$

$$\frac{x(x+6)}{1} \left(\frac{4}{x} + \frac{4}{x+6} \right) = 1 \cdot \frac{x(x+6)}{1}$$

$$\frac{x(x+6)}{1} \cdot \frac{4}{x} + \frac{x(x+6)}{1} \cdot \frac{4}{x+6} = 1 \cdot \frac{x(x+6)}{1}$$

$$4(x+6) + 4x = x(x+6)$$

$$4x + 24 + 4x = x^2 + 6x$$

$$8x + 24 = x^2 + 6x$$

$$\underline{-8x} \qquad \underline{-8x}$$

$$6 = x^2 - 2x$$

$$\underline{-24} \qquad \underline{-24}$$

$$0 = x^2 - 2x - 24$$

$$0 = (x+4)(x-6)$$

$$x+4=0 \quad \text{or} \quad x-6=0$$

$$\underline{-4} \quad \underline{-4} \qquad \underline{+6} \quad \underline{+6}$$

$$x = -4 \quad \text{or} \quad x = 6$$

↑

Time
cannot be
negative.

Bart takes 6 hours,
and Angel takes
12 hours.

6. Together, Topsy and Turvey can eat a 40 pound bag of dog food in 2 weeks. Topsy, by herself can eat a 40 pound bag of dog food in three weeks less than it takes Turvey to eat the full bag by himself. How many weeks would a 40 pound bag last Turvey?

Let x = time it takes Turvey to finish the
the 40 pound bag

then

= time it takes Topsy to finish the bag

	Rate	Time (WT)	Amount of work done
Topsy			
Turvey			
Total			

Answer: 5. Bart takes 6 hours, and Angel takes 12 hours

7. A rowing team can row 45 miles upstream and back again in 8 hours. If the speed of the current is 3 mph, what is the speed of the team's outrigger in still water?

Let x = speed of outrigger in still water

	Rate	Time	Distance
upstream	$x - 3$	$\frac{45}{x - 3}$	45
downstream	$x + 3$	$\frac{45}{x + 3}$	45

$$\frac{(x-3)(x+3)}{1} \left(\frac{45}{x-3} + \frac{45}{x+3} \right) = 8 \cdot \frac{(x-3)(x+3)}{1}$$

$$45(x+3) + 45(x-3) = 8(x^2 - 9)$$

$$45x + 135 + 45x - 135 = 8x^2 - 72$$

$$90x = 8x^2 - 72$$

$$\begin{array}{r} -90x \\ 0 = 8x^2 - 90x - 72 \end{array}$$

$$\begin{array}{r} -90x \\ 0 = 8x^2 - 90x - 72 \end{array}$$

$$\frac{0}{2} = \frac{8x^2 - 90x - 72}{2}$$

$$0 = 4x^2 - 45x - 36$$

$$0 = 4x^2 + 3x - 48x - 36$$

$$0 = (4x + 3)(x - 12)$$

$$4x + 3 = 0 \quad \text{or} \quad x - 12 = 0$$

$$\begin{array}{r} -3 \quad -3 \\ 4x = -3 \quad \text{or} \quad x = 12 \end{array}$$

$$\frac{4x}{4} = \frac{-3}{4}$$

$$x = -\frac{3}{4}$$

↑

Speed

cannot be

negative.

Product	
-1	144
-2	72
-3	48
3	-48
Sum	-45

$$4x \quad 3$$

x	$4x^2$	$3x$
-12	$-48x$	36

The speed of the outrigger is 12 mph.

8. A cyclist traveled 80 miles into a headwind, then returned with the advantage of a tailwind to the starting point in 9 hours. If the speed of the wind was 2 miles per hour, what was the speed of the cyclist with no wind?

Let x = speed of cyclist with no wind

	Rate	Time	Distance
headwind			
tailwind			

Answer: 7. The speed of the outrigger is 12 mph.