

6.1 Inverse Functions

Solutions

If $\{-2, -1, 0, 1, 2\}$ is the domain of each function, find the range.

1. $f(x) = x^2 + x - 1$

x	$f(x)$
-2	$(-2)^2 + (-2) - 1 = 4 - 2 - 1 = \mathbf{1}$
-1	$(-1)^2 + (-1) - 1 = 1 - 1 - 1 = \mathbf{-1}$
0	$0^2 + 0 - 1 = \mathbf{-1}$
1	$1^2 + 1 - 1 = 1 + 1 - 1 = \mathbf{1}$
2	$2^2 + 2 - 1 = 4 + 2 - 1 = \mathbf{5}$

Range = $\{-1, 1, 5\}$

2.

3. $f(x) = 2x^2 + x$

x	$f(x)$
-2	$2(-2)^2 + (-2) = 2 \cdot 4 - 2 = \mathbf{6}$
-1	$2(-1)^2 + (-1) = 2 \cdot 1 - 1 = \mathbf{1}$
0	$2 \cdot 0^2 + 0 = \mathbf{0}$
1	$2 \cdot 1^2 + 1 = 2 \cdot 1 + 1 = \mathbf{3}$
2	$2 \cdot 2^2 + 2 = 2 \cdot 4 + 2 = \mathbf{10}$

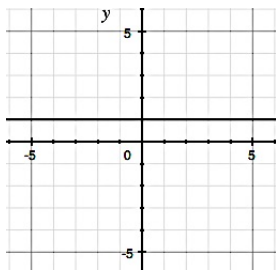
Range = $\{6, 1, 0, 3, 10\}$

4. $f(x) = 3x^2 - 2x$

Answers: 1. $\{-1, 1, 5\}$; 3. $\{6, 1, 0, 3, 10\}$

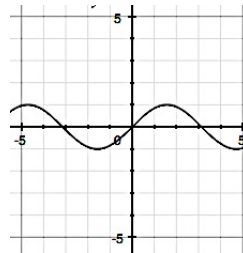
Determine if the following graphs are of relations, functions, or one-to-one functions.

5.

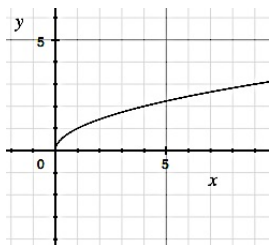


Passes the vertical line test, so it is a **function**.

6.

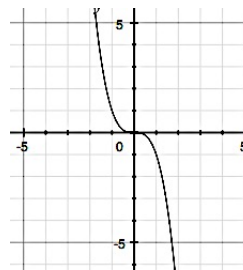


7.

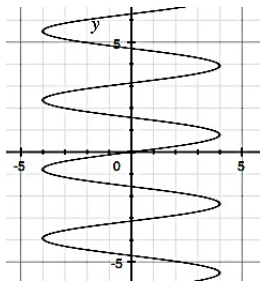


Passes the vertical line test and the horizontal line test, so it is a **1-1 function**.

8.

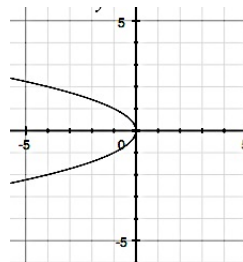


9.

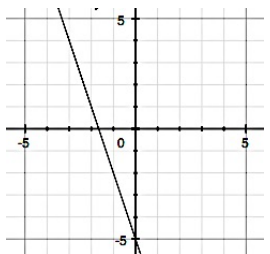


Fails the vertical line test, so it is a **relation**.

10.

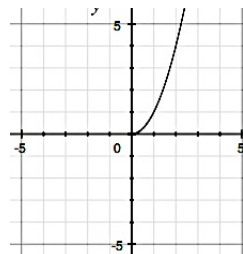


11.



Passes the vertical line test and the horizontal line test, so it is a **1-1 function**.

12.



Answers: 5. function; 7. one-to-one function; 9. relation; 11. one-to-one function

Determine the inverse of the following functions.

13. $f(x) = 2x + 7$

$$y = 2x + 7$$

$$x = 2y + 7$$

$$\underline{-7} \quad \underline{-7}$$

$$x - 7 = 2y$$

$$\frac{x-7}{2} = \frac{2y}{2}$$

$$y = \frac{x-7}{2}$$

$$f^{-1}(x) = \frac{x-7}{2}$$

14. $f(x) = 3x - 1$

15. $f(x) = \frac{1}{2x-3}$

$$y = \frac{1}{2x-3}$$

$$x = \frac{1}{2y-3}$$

$$(2y-3)x = \frac{1}{2y-3}(2y-3)$$

$$2xy - 3x = 1$$

$$\underline{+3x} \quad \underline{+3x}$$

$$2xy = 3x + 1$$

$$\frac{2xy}{2x} = \frac{3x+1}{2x}$$

$$y = \frac{3x+1}{2x}$$

$$f^{-1}(x) = \frac{3x+1}{2x}$$

16. $f(x) = \frac{1}{5x+2}$

Answers: 13. $f^{-1}(x) = \frac{x-7}{2}$; 15. $f^{-1}(x) = \frac{3x+1}{2x}$

Determine the inverse of the following functions.

17. $f(x) = \frac{x}{2x-5}$

$$y = \frac{x}{2x-5}$$

$$x = \frac{y}{2y-5}$$

$$(2y-5)x = \frac{y}{2y-5}(2y-5)$$

$$2xy - 5x = y$$

$$\quad \quad \quad \underline{+5x} \quad \underline{+5x}$$

$$2xy = 5x + y$$

$$\quad \quad \quad \underline{-y} \quad \quad \underline{-y}$$

$$2xy - y = 5x$$

$$y(2x - 1) = 5x$$

$$\frac{y(2x-1)}{2x-1} = \frac{5x}{2x-1}$$

$$y = \frac{5x}{2x-1}$$

$$f^{-1}(x) = \frac{5x}{2x-1}$$

18. $f(x) = \frac{x+2}{x-4}$

Answer: 17. $f^{-1}(x) = \frac{5x}{2x-1}$