

6.2 Exponential Expressions and Equations

Solutions

Evaluate each expression for the given value.	
1. 3^x for $x = -2$ $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$	2. 2^x for $x = -3$
3. 27^x for $x = \frac{1}{3}$ $27^{\frac{1}{3}} = (3^3)^{\frac{1}{3}} = 3^{3 \cdot \frac{1}{3}} = 3$	4. 81^x for $x = \frac{1}{2}$
5. $\left(\frac{1}{4}\right)^x$ for $x = \frac{1}{2}$ $\left(\frac{1}{4}\right)^{\frac{1}{2}} = \frac{1^{\frac{1}{2}}}{4^{\frac{1}{2}}} = \frac{1}{(2^2)^{\frac{1}{2}}} = \frac{1}{2}$	6. $\left(\frac{1}{8}\right)^x$ for $x = \frac{1}{3}$
7. $\left(\frac{1}{4}\right)^x$ for $x = -\frac{1}{2}$ $\left(\frac{1}{4}\right)^{-\frac{1}{2}} = \left(\frac{4}{1}\right)^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = 2$	8. $\left(\frac{1}{8}\right)^x$ for $x = -\frac{1}{3}$
Answers: 1. $\frac{1}{9}$; 3. 3; 5. $\frac{1}{2}$; 7. 2	

Solve for x .	
<p>9. $25 = 5^x$</p> $5^2 = 5^x$ $2 = x$ $x = \boxed{2}$	<p>10. $81 = 3^x$</p>
<p>11. $64 = 2^x$</p> $2^6 = 2^x$ $6 = x$ $x = \boxed{6}$	<p>12. $27 = 3^x$</p> $\begin{array}{r} 2 \overline{)64} \\ 2 \overline{)32} \\ 2 \overline{)16} \\ 2 \overline{)8} \\ 2 \overline{)4} \\ \quad 2 \end{array}$
<p>13. $25^x = 5$</p> $(5^2)^x = 5^1$ $5^{2x} = 5^1$ $2x = 1$ $\frac{2x}{2} = \frac{1}{2}$ $x = \boxed{\frac{1}{2}}$	<p>14. $81^x = 3$</p>
<p>15. $\frac{1}{25} = 5^x$</p> $\frac{1}{5^2} = 5^x$ $5^{-2} = 5^x$ $-2 = x$ $x = \boxed{-2}$	<p>16. $\frac{1}{81} = 3^x$</p>
<p>Answers: 9. 2; 11. 6; 13. $\frac{1}{2}$; 15. -2</p>	

Solve for x .

17. $4^x = 32$

$$(2^2)^x = 2^5$$

$$2^{2x} = 2^5$$

$$2x = 5$$

$$\frac{2x}{2} = \frac{5}{2}$$

$$x = \boxed{\frac{5}{2}}$$

$$\begin{array}{r} 2 \overline{)32} \\ 2 \overline{)16} \\ 2 \overline{)8} \\ 2 \overline{)4} \\ 2 \end{array}$$

18. $27^x = 9$

19. $16^{x-1} = 4$

$$(2^4)^{(x-1)} = 2^2$$

$$2^{4(x-1)} = 2^2$$

$$4x - 4 = 2$$

$$\color{red}{+4} \quad \color{red}{+4}$$

$$4x = 6$$

$$\frac{4x}{4} = \frac{6}{4}$$

$$x = \boxed{\frac{3}{2}}$$

$$\begin{array}{r} 2 \overline{)16} \\ 2 \overline{)8} \\ 2 \overline{)4} \\ 2 \end{array}$$

20. $27^{x+1} = 81$

21. $5^{2x} = \frac{1}{25}$

$$5^{2x} = \frac{1}{5^2}$$

$$5^{2x} = 5^{-2}$$

$$2x = -2$$

$$\frac{2x}{2} = \frac{-2}{2}$$

$$x = \boxed{-1}$$

22. $3^{4x} = \frac{1}{27}$

Answers: 17. $\frac{5}{2}$; 19. $\frac{3}{2}$; 21. -1

Solve for x .

$$\begin{aligned} 23. \quad \left(\frac{1}{2}\right)^x &= 16 \\ (2^{-1})^x &= 2^4 \\ 2^{-x} &= 2^4 \\ -x &= 4 \\ -1(-x) &= -1 \cdot 4 \\ x &= \boxed{-4} \end{aligned}$$

$$\begin{array}{r} 2 \overline{)16} \\ 2 \overline{)8} \\ 2 \overline{)4} \\ \quad 2 \end{array}$$

$$24. \quad \left(\frac{1}{9}\right)^{x+1} = 27$$

$$\begin{aligned} 25. \quad 4^{2x} &= 8^{x-1} \\ (2^2)^{2x} &= (2^3)^{x-1} \\ 2^{4x} &= 2^{3(x-1)} \\ 4x &= 3x - 3 \\ \underline{-3x} \quad \underline{-3x} & \\ x &= -3 \\ x &= \boxed{-3} \end{aligned}$$

$$26. \quad 9^{2x-4} = 3^{x+3}$$

Answers: 23. -4; 25. -3

Use the compound interest formula to find the accumulation amount for each of the following scenarios.

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

- 27.** Principal: \$2,000
Annual rate: 4.6%
Number of years: 3
Compound: yearly

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A = 2000 \left(1 + \frac{.046}{1} \right)^3$$

$$A = 2000(1.046)^3$$

$$A = 2,288.890672$$

$$A = \boxed{\$2,288.89}$$

- 28.** Principal: \$3,500
Annual rate: 2.7%
Number of years: 4
Compound: monthly

- 29.** Principal: \$4,600
Annual rate: 3.8%
Number of years: 1
Compound: quarterly

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A = 4600 \left(1 + \frac{.038}{4} \right)^{4 \cdot 1}$$

$$A = 4600(1.0095)^4$$

$$A = 4777.306713$$

$$A = \boxed{\$4,777.31}$$

- 30.** Principal: \$2,500
Annual rate: 5.3%
Number of years: 2
Compound: semi-annually

Answers: **27.** \$2,288.89; **29.** \$4,777.31