

6.5 Properties of Logarithms

Solutions

Use the properties of logarithms to rewrite the expressions as logarithms of individual numbers or variables. Simplify where possible.

1. $\log_b(5x)$

$$= \log_b 5 + \log_b x$$

2. $\log_b(4y)$

3. $\log_b(x^2)$

$$= 2 \cdot \log_b x$$

4. $\log_b(w^3)$

5. $\log_{10}(10w)$

$$= \log_{10} 10 + \log_{10} w$$

$$= 1 + \log w$$

6. $\log_5(25y)$

7. $\log_b\left(\frac{1}{2}\right)$

$$= \log_b 1 - \log_b 2$$

$$= 0 - \log_b 2$$

$$= -\log_b 2$$

8. $\log_b\left(\frac{1}{x}\right)$

Answers: 1. $\log_b 5 + \log_b x$; 3. $2\log_b x$; 5. $1 + \log w$; 7. $-\log_b 2$

Use the properties of logarithms to rewrite the expressions as logarithms of individual numbers or variables. Simplify where possible.

$$\begin{aligned}
 \mathbf{9.} \quad & \log_b \left(\frac{1}{xy} \right) \\
 &= \log_b 1 - (\log_b x + \log_b y) \\
 &= 0 - (\log_b x + \log_b y) \\
 &= -\log_b x - \log_b y
 \end{aligned}$$

$$\mathbf{10.} \quad \log_b \left(\frac{1}{3y} \right)$$

$$\begin{aligned}
 \mathbf{11.} \quad & \log_b \left(\frac{x^2 y}{5} \right) \\
 &= \log_b x^2 y - \log_b 5 \\
 &= \log_b x^2 + \log_b y - \log_b 5 \\
 &= 2\log_b x + \log_b y - \log_b 5
 \end{aligned}$$

$$\mathbf{12.} \quad \log_b \left(\frac{4y^3}{x} \right)$$

$$\begin{aligned}
 \mathbf{13.} \quad & \log_2 (2\sqrt{5}) \\
 &= \log_2 2 + \log_2 \sqrt{5} \\
 &= 1 + \log_2 5^{\frac{1}{2}} \\
 &= 1 + \frac{1}{2} \log_2 5
 \end{aligned}$$

$$\mathbf{14.} \quad \log_3 (27\sqrt{2})$$

$$\begin{aligned}
 \mathbf{15.} \quad & \log_b \left(\frac{\sqrt{5}}{b} \right) \\
 &= \log_b \sqrt{5} - \log_b b \\
 &= \log_b 5^{\frac{1}{2}} - 1 \\
 &= \frac{1}{2} \log_b 5 - 1
 \end{aligned}$$

$$\mathbf{16.} \quad \log_b \left(\frac{w}{b^5} \right)$$

Answers: **9.** $-\log_b x - \log_b y$; **11.** $2\log_b x + \log_b y - \log_b 5$; **13.** $1 + \frac{1}{2} \log_2 5$; **15.** $\frac{1}{2} \log_b 5 - 1$

Use the properties of logarithms to rewrite the expressions as single logarithms. Simplify if possible.

17. $\log_b 8 + \log_b x$

$$= \log_b (8x)$$

18. $\log_b v + \log_b w$

19. $\frac{1}{2} \log_b (9)$

$$= \log_b 9^{\frac{1}{2}}$$

$$= \log_b (3^2)^{\frac{1}{2}}$$

$$= \log_b 3^{2 \cdot \frac{1}{2}}$$

$$= \log_b 3$$

20. $\frac{1}{5} \log_b (32)$

21. $2\log_b x + 3\log_b (2y)$

$$= \log_b x^2 + \log_b (2y)^3$$

$$= \log_b x^2 + \log_b 8y^3$$

$$= \log_b 8x^2y^3$$

22. $4\log_b (2v) + \log_b (5w)$

23. $2\log_b (3xy) - \log_b (x)$

$$= \log_b (3xy)^2 - \log_b (x)$$

$$= \log_b 9x^2y^2 - \log_b x$$

$$= \log_b \frac{9x^2y^2}{x}$$

$$= \log_b 9xy^2$$

24. $\log_b (3vw) - 2\log_b (w)$

25. $\log_b (2x + 3) - \log_b (5x - 1)$

$$= \log_b \frac{2x + 3}{5x - 1}$$

26. $\log_b (y - 3) - \log_b (5y - 3)$

Answers: 17. $\log_b (8x)$; 19. $\log_b 3$; 21. $\log_b (8x^2y^3)$; 23. $\log_b (9xy^2)$; 25. $\log_b \left(\frac{2x + 3}{5x - 1} \right)$