

6.6 Solving Exponential and Logarithmic Equations

Solutions

Solve.

1. $\log_5(5x + 10) = 2$

$$5^2 = 5x + 10$$

$$25 = 5x + 10$$

$$\underline{-10} \quad \underline{-10}$$

$$15 = 5x$$

$$\frac{15}{5} = \frac{5x}{5}$$

$$3 = x$$

Check: For $x = 3$, $5x + 10 = 15 + 10 = 25$

Solution set: $\{3\}$

2. $\log_3(2x + 1) = 3$

3. $\log(3x - 2) = 2$

$$\log_{10}(3x - 2) = 2$$

$$10^2 = 3x - 2$$

$$100 = 3x - 2$$

$$\underline{+2} \quad \underline{+2}$$

$$102 = 3x$$

$$\frac{102}{3} = \frac{3x}{3}$$

$$34 = x$$

Check: For $x = 34$: $3x - 2 = 100$

Solution set: $\{34\}$

4. $\log(10x + 20) = 3$

Answers: 1. $\{3\}$; 3. $\{34\}$

Solve.

5. $\log_{10}(w^2) = 2$

$$10^2 = w^2$$

$$100 = w^2$$

$$\pm\sqrt{100} = \sqrt{w^2}$$

$$\pm 10 = w$$

Argument Check: For $w = 10$, $w^2 = 100$ ☺

for $w = -10$, $w^2 = 100$ ☺

Solution set: $\{10, -10\}$

6. $\log_5(y^2 - 2y + 1) = 2$

Solve. Round your answers to 4 decimal places (x.xxxx)

7. $4^x = 9$

$$\log 4^x = \log 9$$

$$x \log 4 = \log 9$$

$$\frac{x \log 4}{\log 4} = \frac{\log 9}{\log 4}$$

$$x \approx 1.584962501$$

$$x \approx 1.5850$$

8. $3^x = 5$

Answers: 5. $\{\pm 10\}$; 7. $\{\approx 1.5850\}$

Solve. Round your answers to 4 decimal places (x.xxxx)

9. $3^{x-1} = 2$

$$\log 3^{x-1} = \log 2$$

$$(x-1) \log 3 = \log 2$$

$$\frac{(x-1)\log 3}{\log 3} = \frac{\log 2}{\log 3}$$

$$x-1 = \frac{\log 2}{\log 3}$$

$$\underline{+1} \quad \underline{+1}$$

$$x = \frac{\log 2}{\log 3} + 1$$

$$x \approx 1.630929754$$

$$x \approx \boxed{1.6309}$$

10. $2^{2x+1} = 5$

11. $e^x = 5$

$$\ln e^x = \ln 5$$

$$x \ln e = \ln 5$$

$$x \cdot 1 = \ln 5$$

$$x = \ln 5$$

$$x \approx 1.609437912$$

$$x \approx \boxed{1.6094}$$

12. $e^{2x+1} = 0.04$

Answers: 9. $\{\approx 1.6309\}$; 11. $\{\approx 1.6094\}$

Solve.

13. $\log_2 x = \log_2 7$

$$x = 7$$

Argument Check: For $x = 7$, $x = 7$ ✓

Solution set: $\{7\}$

14. $\log_4 (x + 2) = \log_4 8$

15. $\log_7 (x^2 - 2) = \log_7 (x)$

$$x^2 - 2 = x$$

$$\underline{-x} \quad \underline{-x}$$

$$x^2 - x - 2 = 0$$

$$(x + 1)(x - 2) = 0$$

$$x + 1 = 0 \quad \text{or} \quad x - 2 = 0$$

$$\underline{-1} \quad \underline{-1} \quad \underline{+2} \quad \underline{+2}$$

$$x = -1 \quad \text{or} \quad x = 2$$

Argument Check: For $x = -1$, $x^2 - 2 = -1$ ✗

For $x = 2$, $x^2 - 2 = 4 - 2 = 2$ ✓

Solution set: $\{2\}$

Product -2	
-1	2
1	-2
Sum -1	

Answers: 13. $\{7\}$; 15. $\{2\}$

Solve.

17. $\log_7(x-3) = \log_7(6-2x) - \log_7(3x)$

$$\log_7(x-3) = \log_7 \frac{6-2x}{3x}$$

$$x-3 = \frac{6-2x}{3x}$$

$$3x \cdot (x-3) = \frac{6-2x}{3x} \cdot 3x$$

$$3x^2 - 9x = 6 - 2x$$

$$\quad \quad \quad +2x \quad +2x$$

$$3x^2 - 7x = 6$$

$$\quad \quad \quad -6 \quad -6$$

$$3x^2 - 7x - 6 = 0$$

$$3x^2 + 2x - 9x - 6 = 0$$

$$(x-3)(3x+2) = 0$$

$$x-3=0 \text{ or } 3x+2=0$$

$$\quad \quad \quad +3 \quad +3 \quad \quad \quad -2 \quad -2$$

$$x=3 \text{ or } 3x=-2$$

$$\frac{3x}{3} = \frac{-2}{3}$$

$$x = -\frac{2}{3}$$

Product -18	
-1	18
-2	9
2	-9
Sum -7	

	3x	2
x	3x ²	2x
-3	-9x	-6

Argument Check: For $x = 3$, $x - 3 = 0$ ☹

For $x = -\frac{2}{3}$, $x - 3 = -\frac{2}{3} - 3 = -\frac{8}{3}$ ☹

Solution set: \emptyset

18. $\log_6(x-5) + \log_6(x+1) = \log_6(x+19)$

Answers: 17. \emptyset