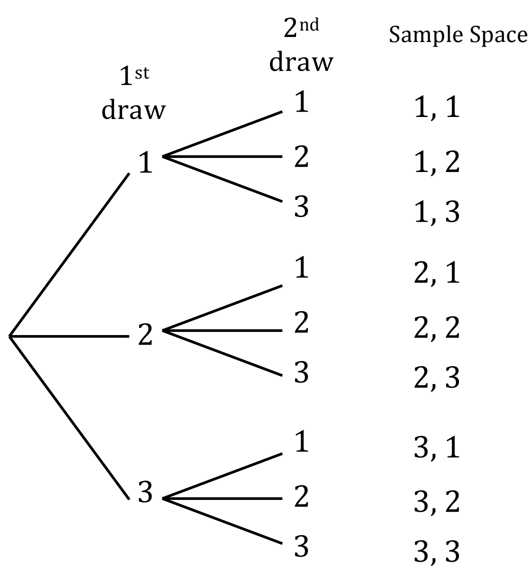
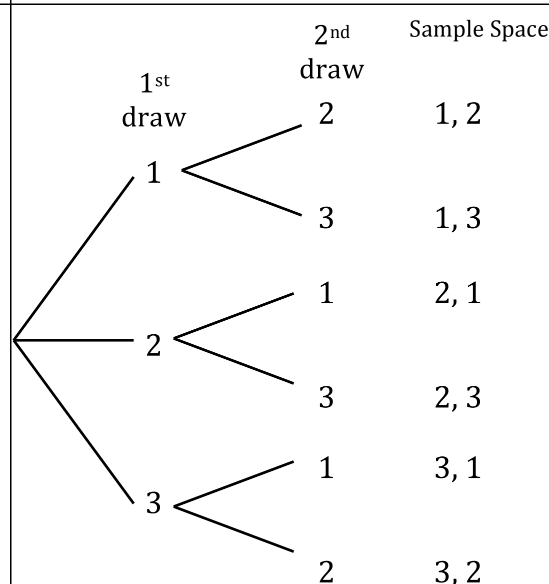
3.2 Independent and Mutually Exclusive Events

Independent Events

Two events, A and B are *independent* if the fact that A occurs does not affect the probability of B occurring.

Let's explore two probability experiments:

In a set of 3 cards, each is marked with a "1", a "2", or a "3". Two cards are chosen randomly. The diagram on the left shows the outcomes if we replace the first card drawn before drawing a second. The diagram on the right shows the outcomes if we don't replace the first card drawn.

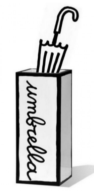
Two cards are drawn WITH replacement (Independent Events)	Two cards are drawn WITHOUT replacement (Dependent Events)
	
Events: A = {1 on 1st draw} B = {2 on 2nd draw}	Events: A = {1 on 1st draw} B = {2 on 2nd draw}
Find the probability of drawing a 1 on the 1st card, and then a 2 on the 2nd card if the 1st card is replaced before drawing the 2nd card. $P(A \text{ and } B) = \dfrac{n(E)}{n(S)} =$	Find the probability of drawing a 1 on the 1st draw and a 2 on the 2nd draw, without replacing the 1st card. $P(A \text{ and } B) = \dfrac{n(E)}{n(S)} =$
Drawing two cards with replacement ***does not*** *affect the probability of the second draw.*	*Drawing two cards without replacement* ***does*** *affect the probability of the second draw.*

Demonstration Problems	*Practice Problems*
Find the following probabilities: **1. (a)** A red, a brown, and a green M&M are placed in a bag. Two of them will be drawn at random with replacement $S =$ {(,); (,); (,); (,); (,); (,); (,); (,); (,)} A = {the first M&M drawn is red} B = {the second M&M drawn is green} $P(A) =$ If the first M&M is red, then $P(B) =$ $P(A \text{ and } B) =$	Find the following probabilities: **1. (b)** A red and a green M&M are placed in a bag. Two of them will be drawn at random with replacement $S =$ {(,); (,); (,); (,)} A = {the first M&M drawn is red} B = {the second M&M drawn is green} $P(A) =$ If the first M&M is red, then $P(B) =$ $P(A \text{ and } B) =$
2. (a) Four cards are numbered 1 through 4. Two cards are drawn at random without replacement. $S =$ {(,); (,); (,); (,); (,); (,); (,); (,); (,); (,); (,); (,)} A = {the first card drawn is a 1} B = {the second card drawn is a 4} $P(A) =$ If the first card is a 1, then $P(B) =$ $P(A \text{ and } B) =$	**2. (b)** Three cards are lettered a, b, c. Two cards are drawn at random without replacement. $S =$ {(,); (,); (,); (,); (,); (,)} A = {the first card drawn is an a} B = {the second card drawn is a b} $P(A) =$ If the first card is a, then $P(B) =$ $P(A \text{ and } B) =$

Answers: **1. (b)** $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$, $P(A \text{ and } B) = \frac{1}{4}$; **2. (b)** $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$, $P(A \text{ and } B) = \frac{1}{4}$;

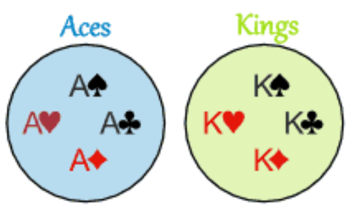
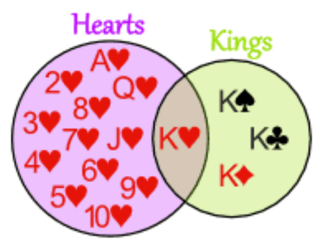
Mutually Exclusive Events

For Eyore, remembering his umbrella and getting caught in the rain are mutually exclusive events.



Two events are *mutually exclusive* if they cannot occur at the same time.

The following Venn diagrams illustrate examples of mutually exclusive and non-mutually exclusive sets.

Mutually Exclusive	Non-mutually Exclusive
A single card is drawn from a standard 52-card deck. The desired outcome is an ace or a king.	*A single card is drawn from a standard 52-card deck. The desired outcome is a heart or a king.*
2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ **9♣** 10♣ J♣ Q♣ 2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ **10♠** J♠ Q♠  2♥ 3♥ 4♥ 5♥ 6♥ 7♥ 8♥ 9♥ 10♥ J♥ Q♥ 2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦	A♣ 2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ **9♣** 10♣ J♣ Q♣ A♠ 2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ **10♠** J♠ Q♠  A♦ 2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦
A = {A♥, A♦, A♠, A♣} B = { K♥, K♦, K♠, K♣}	A={A♥,2♥,3♥,4♥,5♥,6♥,7♥,8♥,9♥, 10♥,J♥,Q♥,K♥} B = { K♥, K♦, K♠, K♣}
Use the formula $\frac{n(E)}{n(S)}$ to find P (A or B) =	Use the formula $\frac{n(E)}{n(S)}$ to find P (A or B) =

Demonstration Problems	*Practice Problems*
Find the following probabilities: A single card is drawn. $S =$ A♣ 2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣ K♣ A♠ 2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠ K♠ A♥ 2♥ 3♥ 4♥ 5♥ 6♥ 7♥ 8♥ 9♥ 10♥ J♥ Q♥ K♥ A♦ 2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦ K♦	Find the following probabilities: A single card is drawn. $S =$ A♣ 2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣ K♣ A♠ 2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠ K♠ A♥ 2♥ 3♥ 4♥ 5♥ 6♥ 7♥ 8♥ 9♥ 10♥ J♥ Q♥ K♥ A♦ 2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦ K♦
3. (a) A = {jacks} B = {kings} $P(A) =$ $P(B) =$ $P(A \text{ or } B) =$	**3. (b)** A = {red cards} B = {spades} $P(A) =$ $P(B) =$ $P(A \text{ or } B) =$
4. (a) A = {face cards} B = {7's} $P(A) =$ $P(B) =$ $P(A \text{ or } B) =$	**4. (b)** A = {prime number cards} B = {kings} $P(A) =$ $P(B) =$ $P(A \text{ or } B) =$
5. (a) A = {even numbered cards} B = {diamonds} $P(A) =$ $P(B) =$ $P(A \text{ or } B) =$	**5. (b)** A = {face cards} B = {red cards} $P(A) =$ $P(B) =$ $P(A \text{ or } B) =$

Answers: **3. (b)** $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{4}$, $P(A \text{ or } B) = \frac{3}{4}$; **4. (b)** $P(A) = \frac{4}{13}$, $P(B) = \frac{1}{13}$, $P(A \text{ or } B) = \frac{5}{13}$;
5. (b) $P(A) = \frac{3}{13}$, $P(B) = \frac{1}{2}$, $P(A \text{ or } B) = \frac{8}{13}$

Demonstration Problems	*Practice Problems*
Two coins are tossed. $S =$ {(,); (,); (,); (,)} **6. (a)** Find the probability that at least one tail occurs. **7. (a)** Find the probability that at exactly one tail occurs.	Two coins are tossed. $S =$ {(,); (,); (,); (,)} **6. (b)** Find the probability that at most one tail occurs. **7. (b)** Find the probability that at exactly one head occurs.
	Answers: **6. (b)** $\frac{3}{4}$; **7. (b)** $\frac{1}{2}$;