

### 3.2 Independent and Mutually Exclusive Events

#### Independent Events

☞ Two events,  $A$  and  $B$  are *independent* if the fact that  $A$  occurs does not affect the probability of  $B$  occurring.



Let's explore two probability experiments:

In a set of 3 cards, each is marked with a "1", a "2", or a "3". Two cards are chosen randomly. The diagram on the left shows the outcomes if we replace the first card drawn before drawing a second. The diagram on the right shows the outcomes if we don't replace the first card drawn.

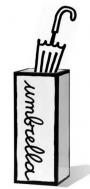
Two cards are drawn WITH replacement (Independent Events)	Two cards are drawn WITHOUT replacement (Dependent Events)
<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>1<sup>st</sup> draw</p> <p>1</p> </div> <div style="text-align: center;"> <p>2<sup>nd</sup> draw</p> <p>1</p> </div> <div style="text-align: center;"> <p>Sample Space</p> <p>1, 1</p> </div> </div> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>1</p> <p>2</p> </div> <div style="text-align: center;"> <p>1</p> <p>2</p> </div> <div style="text-align: center;"> <p>1, 2</p> <p>1, 3</p> </div> </div> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>2</p> <p>3</p> </div> <div style="text-align: center;"> <p>1</p> <p>2</p> <p>3</p> </div> <div style="text-align: center;"> <p>2, 1</p> <p>2, 2</p> <p>2, 3</p> </div> </div> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>3</p> <p>1</p> <p>2</p> <p>3</p> </div> <div style="text-align: center;"> <p>1</p> <p>2</p> <p>3</p> </div> <div style="text-align: center;"> <p>3, 1</p> <p>3, 2</p> <p>3, 3</p> </div> </div>	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>1<sup>st</sup> draw</p> <p>1</p> </div> <div style="text-align: center;"> <p>2<sup>nd</sup> draw</p> <p>2</p> </div> <div style="text-align: center;"> <p>Sample Space</p> <p>1, 2</p> </div> </div> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>1</p> <p>3</p> </div> <div style="text-align: center;"> <p>1</p> <p>3</p> </div> <div style="text-align: center;"> <p>1, 3</p> <p>2, 1</p> </div> </div> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>2</p> <p>3</p> </div> <div style="text-align: center;"> <p>1</p> <p>3</p> </div> <div style="text-align: center;"> <p>2, 1</p> <p>2, 3</p> </div> </div> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>3</p> <p>1</p> <p>2</p> </div> <div style="text-align: center;"> <p>1</p> <p>2</p> </div> <div style="text-align: center;"> <p>3, 1</p> <p>3, 2</p> </div> </div>
<p>Events: <math>A = \{1 \text{ on } 1^{\text{st}} \text{ draw}\}</math>  <math>B = \{2 \text{ on } 2^{\text{nd}} \text{ draw}\}</math></p>	<p>Events: <math>A = \{1 \text{ on } 1^{\text{st}} \text{ draw}\}</math>  <math>B = \{2 \text{ on } 2^{\text{nd}} \text{ draw}\}</math></p>
<p>Find the probability of drawing a 1 on the 1<sup>st</sup> card, and then a 2 on the 2<sup>nd</sup> card if the 1<sup>st</sup> card is replaced before drawing the 2<sup>nd</sup> card.</p> $P(A \text{ and } B) = \frac{n(E)}{n(S)} =$	<p>Find the probability of drawing a 1 on the 1<sup>st</sup> draw and a 2 on the 2<sup>nd</sup> draw, without replacing the 1<sup>st</sup> card.</p> $P(A \text{ and } B) = \frac{n(E)}{n(S)} =$
<p><i>Drawing two cards with replacement <b>does not</b> affect the probability of the second draw.</i></p>	<p><i>Drawing two cards without replacement <b>does</b> affect the probability of the second draw.</i></p>

<i>Demonstration Problems</i>	<i>Practice Problems</i>
<p>Find the following probabilities:</p> <p><b>1. (a)</b> A red, a brown, and a green M&amp;M are placed in a bag. Two of them will be drawn at random with replacement</p> <p><math>S =</math>  <math>\{( \quad , \quad ); ( \quad , \quad ); ( \quad , \quad );</math>  <math>( \quad , \quad ); ( \quad , \quad ); ( \quad , \quad );</math>  <math>( \quad , \quad ); ( \quad , \quad ); ( \quad , \quad )\}</math></p> <p><math>A = \{\text{the first M\&amp;M drawn is red}\}</math>  <math>B = \{\text{the second M\&amp;M drawn is green}\}</math></p> <p><math>P(A) =</math></p> <p><math>P(B   A) =</math></p> <p><math>P(A \text{ and } B) =</math></p> <p><b>2. (a)</b> Four cards are numbered 1 through 4. Two cards are drawn at random without replacement.</p> <p><math>S =</math>  <math>\{( \quad , \quad ); ( \quad , \quad ); ( \quad , \quad );</math>  <math>( \quad , \quad ); ( \quad , \quad ); ( \quad , \quad );</math>  <math>( \quad , \quad ); ( \quad , \quad ); ( \quad , \quad );</math>  <math>( \quad , \quad ); ( \quad , \quad ); ( \quad , \quad )\}</math></p> <p><math>A = \{\text{the first card drawn is a 1}\}</math>  <math>B = \{\text{the second card drawn is a 4}\}</math></p> <p><math>P(A) =</math></p> <p><math>P(B   A) =</math></p> <p><math>P(A \text{ and } B) =</math></p>	<p>Find the following probabilities:</p> <p><b>1. (b)</b> A red and a green M&amp;M are placed in a bag. Two of them will be drawn at random with replacement</p> <p><math>S =</math>  <math>\{( \quad , \quad ); ( \quad , \quad );</math>  <math>( \quad , \quad ); ( \quad , \quad )\}</math></p> <p><math>A = \{\text{the first M\&amp;M drawn is red}\}</math>  <math>B = \{\text{the second M\&amp;M drawn is green}\}</math></p> <p><math>P(A) =</math></p> <p><math>P(B   A) =</math></p> <p><math>P(A \text{ and } B) =</math></p> <p><b>2. (b)</b> Three cards are lettered <math>a, b, c</math>. Two cards are drawn at random without replacement.</p> <p><math>S =</math>  <math>\{( \quad , \quad ); ( \quad , \quad );</math>  <math>( \quad , \quad ); ( \quad , \quad );</math>  <math>( \quad , \quad ); ( \quad , \quad )\}</math></p> <p><math>A = \{\text{the first card drawn is an } a\}</math>  <math>B = \{\text{the second card drawn is a } b\}</math></p> <p><math>P(A) =</math></p> <p><math>P(B   A) =</math></p> <p><math>P(A \text{ and } B) =</math></p>
<p>Answers: <b>1. (b)</b> <math>P(A) = \frac{1}{2}, P(B) = \frac{1}{2}, P(A \text{ and } B) = \frac{1}{4}</math>; <b>2. (b)</b> <math>P(A) = \frac{1}{3}, P(B) = \frac{1}{2}, P(A \text{ and } B) = \frac{1}{4}</math></p>	



### Mutually Exclusive Events

For Eyore, remembering his umbrella and getting caught in the rain are mutually exclusive events.



☞ Two events are *mutually exclusive* if they cannot occur at the same time.

The following Venn diagrams illustrate examples of mutually exclusive and non-mutually exclusive sets.

Mutually Exclusive	Non-mutually Exclusive
<p><i>A single card is drawn from a standard 52-card deck. The desired outcome is an ace or a king.</i></p>	<p><i>A single card is drawn from a standard 52-card deck. The desired outcome is a heart or a king.</i></p>
<p>2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣                  2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>Aces</p> </div> <div style="text-align: center;"> <p>Kings</p> </div> </div> <p>2♥ 3♥ 4♥ 5♥ 6♥ 7♥ 8♥ 9♥ 10♥ J♥ Q♥                  2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦</p>	<p>A♣ 2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣                  A♠ 2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>Hearts</p> </div> <div style="text-align: center;"> <p>Kings</p> </div> </div> <p>A♦ 2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦</p>
<p><math>A = \{A♥, A♦, A♠, A♣\}</math>  <math>B = \{K♥, K♦, K♠, K♣\}</math></p>	<p><math>A = \{A♥, 2♥, 3♥, 4♥, 5♥, 6♥, 7♥, 8♥, 9♥, 10♥, J♥, Q♥, K♥\}</math>  <math>B = \{K♥, K♦, K♠, K♣\}</math></p>
<p>Use the formula <math>\frac{n(E)}{n(S)}</math> to find  <math>P(A \text{ or } B) =</math></p>	<p>Use the formula <math>\frac{n(E)}{n(S)}</math> to find  <math>P(A \text{ or } B) =</math></p>

<i>Demonstration Problems</i>	<i>Practice Problems</i>
<p>Find the following probabilities: A single card is drawn. <math>S =</math>  <math>A \clubsuit 2 \clubsuit 3 \clubsuit 4 \clubsuit 5 \clubsuit 6 \clubsuit 7 \clubsuit 8 \clubsuit 9 \clubsuit 10 \clubsuit J \clubsuit Q \clubsuit K \clubsuit</math>  <math>A \spadesuit 2 \spadesuit 3 \spadesuit 4 \spadesuit 5 \spadesuit 6 \spadesuit 7 \spadesuit 8 \spadesuit 9 \spadesuit 10 \spadesuit J \spadesuit Q \spadesuit K \spadesuit</math>  <math>A \heartsuit 2 \heartsuit 3 \heartsuit 4 \heartsuit 5 \heartsuit 6 \heartsuit 7 \heartsuit 8 \heartsuit 9 \heartsuit 10 \heartsuit J \heartsuit Q \heartsuit K \heartsuit</math>  <math>A \diamondsuit 2 \diamondsuit 3 \diamondsuit 4 \diamondsuit 5 \diamondsuit 6 \diamondsuit 7 \diamondsuit 8 \diamondsuit 9 \diamondsuit 10 \diamondsuit J \diamondsuit Q \diamondsuit K \diamondsuit</math></p> <p><b>3. (a)</b>  <math>A = \{\text{jacks}\} \quad B = \{\text{kings}\}</math>  <math>P(A) =</math>  <math>P(B) =</math>  <math>P(A \text{ or } B) =</math></p> <p><b>4. (a)</b>  <math>A = \{\text{face cards}\} \quad B = \{7\text{'s}\}</math>  <math>P(A) =</math>  <math>P(B) =</math>  <math>P(A \text{ or } B) =</math></p> <p><b>5. (a)</b>  <math>A = \{\text{even numbered cards}\}</math>  <math>B = \{\text{diamonds}\}</math>  <math>P(A) =</math>  <math>P(B) =</math>  <math>P(A \text{ or } B) =</math></p>	<p>Find the following probabilities: A single card is drawn. <math>S =</math>  <math>A \clubsuit 2 \clubsuit 3 \clubsuit 4 \clubsuit 5 \clubsuit 6 \clubsuit 7 \clubsuit 8 \clubsuit 9 \clubsuit 10 \clubsuit J \clubsuit Q \clubsuit K \clubsuit</math>  <math>A \spadesuit 2 \spadesuit 3 \spadesuit 4 \spadesuit 5 \spadesuit 6 \spadesuit 7 \spadesuit 8 \spadesuit 9 \spadesuit 10 \spadesuit J \spadesuit Q \spadesuit K \spadesuit</math>  <math>A \heartsuit 2 \heartsuit 3 \heartsuit 4 \heartsuit 5 \heartsuit 6 \heartsuit 7 \heartsuit 8 \heartsuit 9 \heartsuit 10 \heartsuit J \heartsuit Q \heartsuit K \heartsuit</math>  <math>A \diamondsuit 2 \diamondsuit 3 \diamondsuit 4 \diamondsuit 5 \diamondsuit 6 \diamondsuit 7 \diamondsuit 8 \diamondsuit 9 \diamondsuit 10 \diamondsuit J \diamondsuit Q \diamondsuit K \diamondsuit</math></p> <p><b>3. (b)</b>  <math>A = \{\text{red cards}\} \quad B = \{\text{spades}\}</math>  <math>P(A) =</math>  <math>P(B) =</math>  <math>P(A \text{ or } B) =</math></p> <p><b>4. (b)</b>  <math>A = \{\text{prime number cards}\} \quad B = \{\text{kings}\}</math>  <math>P(A) =</math>  <math>P(B) =</math>  <math>P(A \text{ or } B) =</math></p> <p><b>5. (b)</b>  <math>A = \{\text{face cards}\}</math>  <math>B = \{\text{red cards}\}</math>  <math>P(A) =</math>  <math>P(B) =</math>  <math>P(A \text{ or } B) =</math></p>
<p>Answers: <b>3. (b)</b> <math>P(A) = \frac{1}{2}, P(B) = \frac{1}{4}, P(A \text{ or } B) = \frac{3}{4}</math>; <b>4. (b)</b> <math>P(A) = \frac{4}{13}, P(B) = \frac{1}{13}, P(A \text{ or } B) = \frac{5}{13}</math>;  <b>5. (b)</b> <math>P(A) = \frac{3}{13}, P(B) = \frac{1}{2}, P(A \text{ or } B) = \frac{8}{13}</math></p>	

