3.3 Two Basic Rules of Probability

If we want to know the probability of drawing a 2 on the first card and a 3 on the 2nd card from a standard 52-card deck, the diagram would be very large and tedious to draw. Fortunately, we have a formula that we can use to determine the probability of two events instead:

Multiplication Rule	$P(A \text{ and } B) = P(A) \bullet P(B\|A)$

Recall from section 3.1, the following definition and notation for conditional probability:

- The *conditional probability* of an event B with respect to event A is the probability that event B occurs after A has already occurred, denoted $P(B|A)$.

Recall from section 3.2, the probability experiment in which two cards were drawn from a set of cards marked 1, 2, and 3.

A = {the 1st card is a "1"} and B = {the 2nd card is a "2"}

With replacement	Without replacement
$P(A \text{ and } B) = \dfrac{1}{9}$	$P(A \text{ and } B) = \dfrac{1}{6}$

We found these probabilities by counting the number of desired outcomes in the sample space for each. We can instead apply the formula from above.

Independent Events Two cards drawn with replacement	Dependent Events Two cards drawn without replacement
1st card — Sample space $S=\{1, 2, 3\}$ — $P(A) = \dfrac{1}{3}$	1st card — Sample space $S=\{1, 2, 3\}$ — $P(A) = \dfrac{1}{3}$
2nd card — Sample space $S=\{1, 2, 3\}$ — $P(B\|A) = \dfrac{1}{3}$	2nd card — Sample space $S=\{2, 3\}$ — $P(B\|A) = \dfrac{1}{2}$
$P(A \text{ and } B) = P(A) \bullet P(B\|A) = \dfrac{1}{3} \bullet \dfrac{1}{3} = \dfrac{1}{9}$	$P(A \text{ and } B) = P(A) \bullet P(B\|A) = \dfrac{1}{3} \bullet \dfrac{1}{2} = \dfrac{1}{6}$

Notice that for independent events: $P(B|A) = P(B)$

Demonstration Problems	*Practice Problems*
1. (a) Find the probability that a randomly chosen family has exactly 2 sons.	**1. (b)** Find the probability that a student correctly guesses both questions on a two-question true-false quiz.
A coin is flipped and a card is drawn from a standard 52-card deck. A = {head}, B = {tail}, C = {hearts}, D = {queens}	A coin is flipped and a six-sided die is rolled A = {head}, B = {tail}, C = {even numbers}, D = {3}
2. (a) $P(A \text{ and } C) =$	**2. (b)** $P(A \text{ and } C) =$
3. (a) $P(A \text{ and } D) =$	**3. (b)** $P(A \text{ and } D) =$
4. (a) $P(B \text{ and } D) =$	**4. (b)** $P(B \text{ and } D) =$
5. (a) Suppose the probability that an airplane's primary electrical system will work is .99 and the probability that it's secondary back-up system works is .98. Find the probability that both will fail.	**5. (b)** An automobile salesperson finds the probability of making a sale is 0.21. If she talks to 4 customers, find the probability she will make 4 sales.
6. (a) A coin is tossed 5 times. What is the probability of getting at least one tail? Hint: $P(\text{no tails}) + P(\text{at least one tail}) = 1$	**6. (b)** A true-false quiz has 4 questions. What is the probability of correctly guessing at least one question?

Answers: **1. (b)** $\frac{1}{4}$; **2. (b)** $P(A \text{ and } C) = \frac{1}{4}$; **3. (b)** $P(A \text{ and } D) = \frac{1}{12}$; **4. (b)** $P(B \text{ and } D) = \frac{1}{12}$; **5. (b)** ≈ 0.002; **6. (b)** $\frac{15}{16}$

Demonstration Problems	*Practice Problems*
Two cards are drawn randomly from a standard 52-card deck without replacement. A = {kings} B = {hearts} C = {black cards} Let A_1 denote "A on the first card" and A_2 denote "A on the second card". We will use this subscript notation for sets B and C as well.	Two cards are drawn randomly from a standard 52-card deck without replacement. A = {aces} B = {queens} C = {red cards} Let A_1 denote "A on the first card" and A_2 denote "A on the second card". We will use this subscript notation for sets B and C as well.
7. (a) $P(A_2 \mid A_1) =$	**7. (b)** $P(B_2 \mid A_1) =$
8. (a) $P(A_1 \text{ and } A_2) =$	**8. (b)** $P(A_1 \text{ and } B_2) =$
9. (a) $P(B_2 \mid B_1) =$	**9. (b)** $P(C_2 \mid C_1) =$
10. (a) $P(B_1 \text{ and } B_2) =$	**10. (b)** $P(C_1 \text{ and } C_2) =$
	Answers: **7. (b)** $\frac{4}{51}$; **8. (b)** $\frac{4}{663}$; **9. (b)** $\frac{25}{51}$; **10. (b)** $\frac{25}{102}$

Older sister Gabriela Salgueiro was born on Dec. 31, 2013, at 11:52 p.m., weighing 6 pounds, 6 ounces.
Younger twin Sophia Salgueiro was born on Jan. 1, 2014, at 12:00:38 a.m., weighing 5 pounds, 13 ounces.

What is the probability of identical twins having birthdays in separate years?

Although many factors influence the timing of birthdates, we can find an approximate probability by using the following information.

1. The probability of conceiving twins is 1/30.
2. Full-term twins are usually born within minutes of each other, but could be born up to an hour apart.

Now, let's recall the example from section 3.2 in which a single card was drawn from a standard deck of 52-cards.

Let A = {aces}, B = {kings}, C = {hearts}

By counting cards in the sample spaces, we found that

$$P(A \text{ or } B) = \frac{2}{13} \quad \text{and} \quad P(B \text{ or } C) = \frac{4}{13}$$

When the sample space is very large, listing all outcomes of the event space and sample space to find the probability of an event could be unreasonable at best, impossible at worst. Fortunately, again, we have a formula we can use instead.

Addition Rule	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Returning to our examples above:

Mutually exclusive events:	Non-mutually exclusive events:
A♣ 2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣ K♣ A♠ 2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠ K♠ A♥ 2♥ 3♥ 4♥ 5♥ 6♥ 7♥ 8♥ 9♥ 10♥ J♥ Q♥ K♥ A♦ 2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦ K♦	A♣ 2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣ K♣ A♠ 2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠ K♠ A♥ 2♥ 3♥ 4♥ 5♥ 6♥ 7♥ 8♥ 9♥ 10♥ J♥ Q♥ K♥ A♦ 2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦ K♦
Sample space A ={ A♥, A♦, A♠, A♣} $\quad P(A) = \frac{4}{52} = \frac{1}{13}$	Sample space B = { K♥, K♦, K♠, K♣} $\quad P(B) = \frac{4}{52} = \frac{1}{13}$
Sample space B = { K♥, K♦, K♠, K♣} $\quad P(B) = \frac{4}{52} = \frac{1}{13}$	Sample space C={A♥,2♥,3♥,4♥,5♥,6♥, 7♥,8♥,9♥,10♥,J♥,Q♥,K♥} $\quad P(C) = \frac{13}{52} = \frac{4}{13}$
$P(A \text{ and } B) = 0$	$P(B \text{ and } C) = \frac{1}{52}$
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) =$ $\frac{4}{52} + \frac{4}{52} - 0 = \frac{8}{52} = \frac{2}{13}$	$P(B \text{ or } C) = P(B) + P(C) - P(B \text{ and } C) =$ $\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

Notice that for mutually exclusive events, $P(A \text{ and } B) = 0$.

Demonstration Problems	*Practice Problems*
A single card is drawn randomly from a standard 52-card deck. A = {kings} B = {queens} C = {hearts}	A single card is drawn randomly from a standard 52-card deck. A = {aces} B = {red cards} C = {face cards}
11. (a) $P(A \text{ or } B) =$	**11. (b)** $P(A \text{ or } B) =$
12. (a) $P(A \text{ or } C) =$	**12. (b)** $P(A \text{ or } C) =$
13. (a) $P(B \text{ or } C) =$	**13. (b)** $P(B \text{ or } C) =$
	Answers: **11. (b)** $\frac{7}{13}$; **12. (b)** $\frac{4}{13}$; **13. (b)** $\frac{8}{13}$

Blackjack

A blackjack is a 2-card hand in which one card is a 10, jack, queen, or king and the other card is an ace. Find the probability of being dealt a blackjack.

Let A = { A♥, A♦, A♠, A♣}

Let B = {10♥, 10♦, 10♠, 10♣, J♥, J♦, J♠, J♣, Q♥, Q♦, Q♠, Q♣, K♥, K♦, K♠, K♣}

$P(A_1 \text{ and } B_2) + P(B_1 \text{ and } A_2) =$

$P(A_1) \bullet P(B_2 \mid A_1) + P(B_1) \bullet P(A_2 \mid B_1) =$

A Blackjack dealer must turn his/her second card face up. What is the probability that the dealer has a Blackjack if the face up card is an ace?

Birthday Problem

January	February	March
April	May	June
July	August	September
October	November	December

Find the probability that 2 students have the same birthday in this class.
Hint: $P(\text{no matches}) + P(\text{at least one match}) = 1$