Math 210A, Exam 1, 2009

1. Give an example of each of the following . You do not need to justify your answer.
(a) An element in S_5 of order 6.
(b) A homomorphism from S_n onto $\mathbb{Z}_2$.
(c) An infinite group in which all elements have finite order.

2. Let $\sigma = (25)(134) \in S_7$ and $= (23)(567) \in S_7$.
(a) Find $\gamma \in S_7$ such that $\tau = \gamma\sigma\gamma^{-1}$.
(b) Determine, with explanation, the number of conjugates of σ in S_7.

3. (a) Let $\mathbb{Z}$ be the center of the group G. Prove that if $G/\mathbb{Z}$ is cyclic, then G is abelian.
(b) Let G be a nonabelian group of order pq, where p and q are primes.
Prove that $G \cong \mathrm{Inn}(G)$, where $\mathrm{Inn}(G)$ is the inner automorphism group of G.

4. Let $\varphi: G \to G'$ be a homomorphism.
(a) Prove that if $g \in G$ and g has finite order, then $\circ(\varphi(g))$ divides $\circ(g)$.
(b) Prove that if $(|G|, |G'|) = 1$, then $\varphi(x) = 1$ for all $x \in G$.

5. Let G be a finite group with $K \lhd G$. If $(|K|, [G:K]) = 1$, prove that K is the unique subgroup of G having order $|K|$.

6. Prove that there does not exist a homomorphism from S_3 onto $\mathbb{Z}_3$.

7. Let $H \leq S_n$. Let $K = \{\sigma \in H \mid \sigma \text{ is even}\}$.
(a) Prove that $K \leq S_n$.
(b) Prove that $K \lhd H$.
(c) Prove that if $K \neq H$, then $|K| = \frac{1}{2}|H|$.