

Math 210A, Exam 2, 2009

1. Give an example of each of the following. You do not need to justify your answer.
 - (a) A simple group that is abelian.
 - (b) A Sylow 2-subgroup of A_4 .
 - (c) A p -group that is not abelian.

2.
 - (a) State the Index Factorial Theorem.
 - (b) Prove the Index Factorial Theorem.

3. Let G be a group such that $|G| = p^n$ for some prime p and $n \in \mathbb{N}$.
 - (a) Prove $Z(G) \neq \{e\}$.
 - (b) Prove there exists a normal subgroup H of G such that $|H| = p^{n-1}$.

4.
 - (a) Find all abelian groups, up to isomorphism, of order $200 = 2^3 \cdot 5^2$.
 - (b) Let G be a group of order 200 such that G does not have an element of order 20 but does have an element of order 50. Determine, with justification, the isomorphism class of G .
 - (c) How many elements of order 50 does G have? Justify your response.

5. Let G be a group of order 105.
 - (a) Prove G is not simple.
 - (b) If G has normal Sylow 3-subgroup, prove that G is abelian.