Math 210B Exam 1 Spring 2010

1. *Let R be a commutative ring with unity and $a, b \in R$.*

(a) *Prove that if $(a, b) = (d)$ for some $d \in R$, then d is a greatest common divisor of a and b.*
(b) *Prove that the converse of the above statement is false.*

2. *Prove that all euclidean rings are principal ideal domains.*

3. *Let R be an integral domain, $a \in R$ and $I = (a)$.*

(a) *Prove that if I is maximal, then a is irreducible.*
(b) *Prove that the converse of the above statement is false.*
(c) *Prove that x is a prime element in R[x].*
(d) *Prove that (x) is maximal in R[x] if and only if R is a field.*

4. *Let R be a commutative ring with unity and let I be a proper ideal in R. Recall that a ring is called local if it has a unique maximal ideal. (For this problem you may assume that a proper ideal is always contained in some maximal ideal.)*

(a) *Prove that if R is a local ring, then R/I is a local ring.*
$\therefore$ R/I is a local ring.
(b) *Let $M = \{r \in R | r \notin U(R)\}$. Prove that if M is an ideal in R, then R is a local ring.*

5. *Let R be a unique factorization domain and $a, b, c \in R$. Prove that if a and b are relatively prime and $a|bc$, then $a|c$.*

6. *Let R be an integral domain, $Q = Frac(R)$ and $f(x) \in R[x]$ such that $\deg f(x) > 0$.*

(a) *Prove that if f (x) is irreducible in R[x], then f (x) is primitive.*
(b) Prove that the converse of the above statement is false.
(c) *Prove that if f (x) is primitive and irreducible in Q[x], then f (x) is irreducible over R[x].*