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Midterm Exam 1.1 - Math 230A
Fall 2010

1. Let S be an ordered set and $A, B \subset S$ be bounded above.

(a) Write the definition of an upper bound for A.
(b) Write the definition of a least upper bound for A.
(c) If $S = \mathbb{R}$ show that

$$\sup(A+B) = \sup A + \sup B\,.$$

(d) Give an example of an ordered set S and $A \subset S$ which is bounded above, but does not have a least upper bound.

2. Let F be an ordered field.
(a) Write the two axioms which relate the order to the algebraical operations.
(b) Show that if $x > 0$ then $1/x > 0$.
(c) Show that if $x < 0$ and $y < z$ then $xy > xz$.

3. State and prove the Archimedean property of $\mathbb{R}$.

4. State Young's, Hölder's and Minkowski's inequalities and show the proof of Minkowski's inequality. Explain the importance of Minkowski's inequality in our context - one sentence only .

5. Show that for $\mathbf{x}$, $\mathbf{y} \in \mathbb{R}^n$ we have

$$\Big||\mathbf{x}| - |\mathbf{y}|\Big| \leq |\mathbf{x} - \mathbf{y}|\,.$$

Is this true for the p-norm $|\cdot|_p$ in the case of any $p \geq 1$?

6. Let $f : X \to Y$, $g : Y \to X$ and $A_1, A_2 \subset X$ and $B_1, B_2 \subset Y$.
(a) Prove that $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$.

(b) Give an example showing that $f(A_1 \cap A_2) \neq f(A_1) \cap f(A_2)$.

(c) Show that if f and g are onto, then $f \circ g$ is onto.

7. State the definition of a countable set and prove that $\mathbb{Z}$ is countable.