Math 241 Exam 1 9/29/11

1. Write the defintions of the following objects.

(a) A metric space.
(b) A normed space.
(c) A Banach space.
(d) A Hamel basis of a vector space.
(e) A Schauder basis of a normed space.
(f) $(l^p, \|\bullet\|_p)$ Banach space.
(g) $(\mathscr{C}[a, b], \|\bullet\|)$ Banach space.
(h) span (M) where M is a subset of a vector space X.
(i) The dimension of a Banach space.

2. Give examples of the following facts. Explanations with proofs and (or) counterexamples are required.

(a) A normed space which is not a Banach space.
(b) A metric on a vector space which cannot be obtained from a norm.
(c) A Schauder basis which is not a Hamel basis.
(d) An absolutely convergent series which is not convergent.
(e) A Cauchy sequence which is not convergent.

3. Answer the following questions. Explanations with proofs and (or) counterexamples are required.

(a) Is $Y = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 + 2x_2 - 3x_3 = 0\}$ a subspace of $\mathbb{R}^3$?
(b) Are $x_1(t) = t$ and $x_2(t) = t^2$ linearly independent in $\mathscr{C}[0, 1]$?
(c) In a Banach space does $\| x_n \| \to \| x \| \Rightarrow x_n \to x$?
What about $x_n \to x$ implying $\| x_n \| \to \| x \|$?