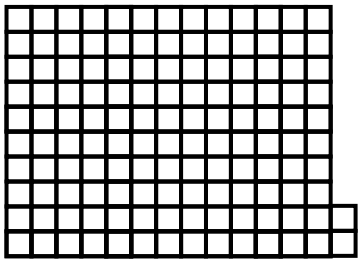
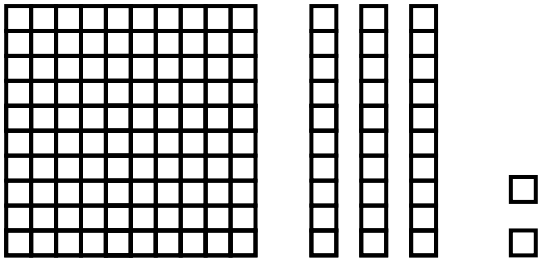
1.1 Introduction to Whole Numbers

To help us communicate large numbers, we use a *place value system*. To describe a large number using a place value system, we state how many of successively larger groups are contained in the number. For example, we can express the number 132 as 1 group of size one hundred, 3 groups of size ten, and 2 groups of size one.

To see how this system is helpful, we will compare Figure A with Figure B. They each contain the same number of small squares of size □.

Figure A	**Figure B**
	
How many small squares make up this figure?	How many small squares make up this set of groupings?
Notice that it is very tedious to count every small square to determine the final number.	If you know the large square is 10×10, then you can quickly determine the number of small squares.

In the number 132, we say 1 is in the *hundreds place value*, 3 is in the *tens place value* and 2 is in the *ones place value.* Notice that each successively larger place value is 10 times the previous.

Another way to model place value is by counting money. Suppose a person wants to withdraw $374 from a bank account. Instead of counting out 374 one dollar bills, the teller is likely to give the person the following:

Three $100 bills
$3 \times \$100$
$300

Seven $10 bills
$7 \times \$10$
$70

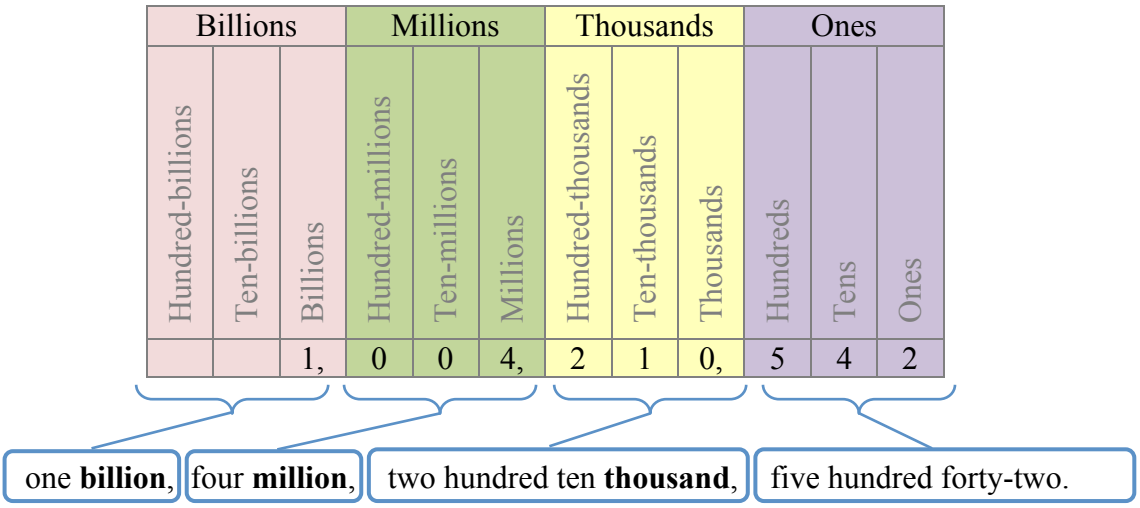
Four $1 bills
$4 \times \$1$
$4

For a very large number such as, 1,004,210,542 we can place an imaginary place value system over the number to understand how many of each place value are contained in the number as follows:

Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones
		1	0	0	4	2	1	0	5	4	2

Demonstration Problems	*Practice Problems*
Find the place-value of the digit 5 in each whole number. **1. (a)** 541 **2. (a)** 153,200 **3. (a)** 4,331,005	Find the place-value of the digit 1 in each whole number. **1. (b)** 541 **2. (b)** 153,200 **3. (b)** 4, 331,005
4. (a) Name the digit in the hundred-thousands place value of 23,421,052	**4. (b)** Name the digit in the hundreds place value of 23,421,052
	Answers: **1. (b)** ones; **2. (b)** hundred-thousands; **3. (b)** thousands; **4. (b)** 0

To avoid having to name each and every place value in words, we use commas to separate large numbers into *periods*, and then say or write whole numbers by naming the number in each period. For example, 1,004,210,542 is read as



Note that a hyphen is used between the tens value and the ones value.

We say 1,004,210,542 is written in *standard form*.

Demonstration Problems	*Practice Problems*
Write each number in words **5. (a)** 541	Write each number in words **5. (b)** 232
6. (a) 153,200	**6. (b)** 145,300
7. (a) 2,012,331,005	**7. (b)** 6,011, 223,002
8. (a) Write twenty-three million, four hundred twenty-one thousand, fifty-two in standard form	**8. (b)** Write twelve thousand, two hundred fifty-three thousand, one hundred five
Answers: **5. (b)** two hundred thirty-two; **6. (b)** one hundred forty-five thousand three hundred; **7. (b)** six billion, eleven million, two hundred twenty-three thousand, two; **8. (b)** 12,253,105	

Notice that $5672 = 5000 + 600 + 70 + 2 = 5 \times 1{,}000 + 6 \times 100 + 7 \times 10 + 2 \times 1$. Written as a sum in this way, the number can be analyzed more thoroughly. When we write a number as a sum of the values of its digits, it is written in *expanded notation*.

Demonstration Problems	*Practice Problems*
Write each number in expanded notation **9. (a)** 541 **10. (a)** 103,200	Write each number in expanded notation **9. (b)** 232 **10. (b)** 140,300
Write each sum as a number in standard form **11. (a)** $4 \times 1{,}000 + 2 \times 100 + 6 \times 10 + 3 \times 1$ **12. (a)** $1 \times 1{,}000{,}000 + 3 \times 10{,}000 + 5 \times 100 + 10 + 1 \times 1$	Write each sum as a number in standard form **11. (b)** $2 \times 1{,}000 + 5 \times 100 + 4 \times 10 + 2 \times 1$ **12. (a)** $1 \times 1{,}000{,}000 + 3 \times 10{,}000 + 5 \times 100 + 1 \times 10 + 1 \times 1$
Answers: **9. (b)** $2 \times 100 + 3 \times 10 + 2 \times 1$; **10. (b)** $100{,}000 + 4 \times 10{,}000 + 3 \times 100$; **11. (b)** 2,542; **12. (b)** 1,030,511	

Rounding and Estimating

Why do we estimate?

Consider the following:

Population of Red Bluff:	14,105 people
National debt:	$20,599,491,122,065
Shasta College enrollment:	12,731 students

The quantities listed above provide more information than is necessary. It is difficult even to verbalize the amount of national debt, for instance. Sometimes we prefer to communicate estimates rather than actual quantities.

We can estimate the numbers given above as:

Population of Red Bluff:	14,000 people
National debt:	$21,000,000,000,000
Shasta College enrollment:	13,000 students

• Notice that 14,000 < 14,105 < 15,000. That is, 14,105 lies between 14,000 and 15,000. It is clear that 14,105 is closer to 14,000 than it is to 15,000.

• Similarly, 20,000,000,000,000 < 20,599,491,122,065 < 21,000,000,000,000 and 15,882,491,122,065 is closer to 16,000,000,000,000 than it is to 15,000,000,000,000.

• Also 12,000 < 12,731 < 13,000 and 12,731 is closer to 13,000 than it is to 12,000.

We *round* numbers by finding the closest multiple of a given place value to that number. For example:

To round 163 to the nearest ten, we determine whether 163 is closer to 160 or 170, the multiples of ten that are just below and just above 163.

Place a dot approximately where 163 belongs on the line between 160 and 170.

160 170

Is 163 closer to 160 or 170? How did you know?

So then 163 rounded to the nearest ten is _____ .

Let's try another one. Round 1,352 to the nearest hundred.

Place a dot approximately where 1,352 belongs on the line between 1,300 and 1,400.

1,300 1,400

Is 1,352 closer to 1,300 or 1,400? How did you know?

So then 1,352 rounded to the nearest hundred is _____ .

Let's round the national debt to the nearest billion.

Place a dot approximately where 20,599,491,122,065 belongs on the line between 20,599,000,000,000 and 20,600,000,000,000.

20,599,000,000,000 20,600,000,000,000

Is 20,599,491,122,065 closer to 20,599,000,000,000 or 20,600,000,000,000?
So then 20,599,491,122,065 rounded to the nearest hundred is

That was fun, but we don't actually have to draw a number line each time we want to round a number to a given place value. We can apply the following steps instead.

Steps for Rounding a Whole Number to a Given Place Value

Steps	*Example:* Round 168 to the nearest **ten**	*Example:* Round 1,352 to the nearest **hundred**
Step 1: Identify the **digit in the given place value** and the digit immediately to the right of it.	163	1,352
Step 2: If the digit to the right is less than 5, leave the **given place value at its current value**. If the digit to the right is 5 or greater, add 1 to the **digit in the given place value**.	Leave the **6** at its current value	Add 1 to the **3**
Step 3: Zero all the digits to the right of the given place value.	160	1,400

Billions			Millions			Thousands			Ones		
Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones

Demonstration Problems	*Practice Problems*
13. (a) Round 3,976 to the nearest ten.	**13. (b)** Round 2,154 to the nearest ten.
14. (a) Round 15,269 to the nearest thousand.	**14. (b)** Round 46,587 to the nearest thousand.
15. (a) Round 198,273,492,000 to the nearest ten-billion.	**15. (b)** Round 596,372,298,400 to the nearest ten-billion.
	Answers: **13. (b)** 2,450; **14. (b)** 47,000; **15. (b)** 600,000,000,000