

**1.4 Multiplying Whole Numbers and Finding Area of Rectangles**

How would you count how many rolls of dimes there are in this box?

Would you count them individually?

Is there another way to count them?



We could add  $10 + 10 + 10 + 10 + 10$  to get the total number of rolls in the box.

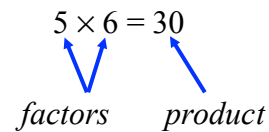
The multiplication  $5 \times 10$  is equivalent to the sum  $10 + 10 + 10 + 10 + 10$ . Multiplication is actually a shorthand notation for repeated addition.

Thus, the number of rolls of dimes is  $5 \times 10 = 10 + 10 + 10 + 10 + 10 = 50$ .

**Vocabulary and Properties**

The result of a multiplication problem is called a *product*.

The numbers being multiplied are called *factors*.



*Notation:* multiplication can be indicated by  $\times$ ,  $\cdot$ , or parentheses

For example:  $2 \times 3$ ,  $2 \cdot 3$ ,  $2(3)$ ,  $(2)3$ , and  $(2)(3)$  all indicate multiplication.

|  |   |
|--|---|
| <p><b><i>Multiplication Property of 0</i></b></p> <p>The product of any number and 0 is 0.<br/>That is, <math>a \cdot 0 = 0</math> and <math>0 \cdot a = 0</math></p>                        | <p><b><i>Multiplication Property of 1</i></b></p> <p>The product of any number and 1 is that number.<br/>That is, <math>a \cdot 1 = a</math> and <math>1 \cdot a = a</math></p>                                     |
| <p><b><i>Commutative Property of Multiplication</i></b></p> <p>Changing the order of factors does not change the product of the factors.<br/>That is, <math>a \cdot b = b \cdot a</math></p> | <p><b><i>Associative Property of Multiplication</i></b></p> <p>Changing the grouping of factors does not change the product of the factors.<br/>That is, <math>(a \cdot b) \cdot c = a \cdot (b \cdot c)</math></p> |
| <p><b><i>Distributive Property</i></b></p> <p>Multiplication distributes over addition.<br/>That is, <math>a(b + c) = a \cdot b + a \cdot c</math></p>                                       |   |

**Multiply:  $231 \times 3$** 

The product  $231 \times 3$  is the same as the repeated addition  $231 + 231 + 231$ . Recall that we can add these numbers by adding the columns in expanded notation:

|     |   |    |   |   |   |     |
|-----|---|----|---|---|---|-----|
| 200 | + | 30 | + | 1 | = | 231 |
| 200 | + | 30 | + | 1 | = | 231 |
| 200 | + | 30 | + | 1 | = | 231 |
|     |   |    |   |   |   |     |

Alternatively, we could use the commutative property, expanded notation, and the distributive property to write:

$$\begin{aligned}
 231 \times 3 &= 3 \times 231 \\
 &= 3 \times (200 + 30 + 1) \\
 &= 3 \times 200 + 3 \times 30 + 3 \times 1 \\
 &= 600 + 90 + 3 \\
 &=
 \end{aligned}$$

Here is a visual “box method”:

|   |     |    |   |   |
|---|-----|----|---|---|
| × | 200 | 30 | 1 |   |
| 3 | 600 | 90 | 3 | ⇒ |

Notice that we can find the product by multiplying each place value by 3. We use the shorthand notation as shown at right:

|   |   |   |
|---|---|---|
| 2 | 3 | 1 |
| × |   | 3 |
|   |   |   |
|   |   |   |

**Consider  $231 \times 5$** 

Let's try the box method:

|   |     |    |   |   |
|---|-----|----|---|---|
| × | 200 | 30 | 1 |   |
| 5 |     |    |   | ⇒ |

And now compare this to the shorthand method:

|   |   |   |
|---|---|---|
| 2 | 3 | 1 |
| × |   | 5 |
|   |   |   |
|   |   |   |

**Consider  $423 \times 25$**

We can use expanded notation in our “box method” to multiply the two numbers and then add the individual products to obtain the final product.

|    |      |     |    |
|----|------|-----|----|
| ×  | 400  | 20  | 3  |
| 20 | 8000 | 400 | 60 |
| 5  | 2000 | 100 | 15 |

⇒

|   |      |
|---|------|
|   | 8000 |
|   | 2000 |
|   | 400  |
|   | 100  |
|   | 60   |
| + | 15   |
|   |      |

We could show the process of multiplying these numbers using expanded notation and the distributive property, but the work would be tedious and difficult to track, so we will begin with our shorthand notation:

|  |   |   |   |   |
|--|---|---|---|---|
|  |   | 4 | 2 | 3 |
|  | × | 2 | 5 |   |
|  |   |   |   |   |
|  |   |   |   |   |
|  |   |   |   |   |

An alternative method for multiplying large numbers is called lattice multiplication. The digits of one number are each multiplied by the digits of the other number and the numbers in the diagonals are added together. The final product is read counterclockwise, beginning with the number at the upper left.

|   |   |   |   |   |   |
|---|---|---|---|---|---|
|   |   | 4 | 2 | 3 | × |
|   | 0 |   | 0 |   | 0 |
| 1 |   | 8 |   | 4 |   |
|   | 2 |   | 1 |   | 1 |
| 0 |   | 0 |   | 0 |   |
|   | 5 |   | 7 |   | 5 |

Thus, we have  $423 \times 25 = 10,575$

Try this one:  $543 \times 36$

Box method:

|   |  |  |  |
|---|--|--|--|
| × |  |  |  |
|   |  |  |  |
|   |  |  |  |

⇒

|   |
|---|
|   |
|   |
|   |
|   |
|   |
| + |
|   |

Shorthand notation:

|          |          |          |          |
|----------|----------|----------|----------|
|          | <b>5</b> | <b>4</b> | <b>3</b> |
| <b>×</b> |          | <b>3</b> | <b>6</b> |
|          |          |          |          |
|          |          |          |          |
|          |          |          |          |
|          |          |          |          |
|          |          |          |          |

Lattice multiplication:

|  |   |   |   |   |   |
|--|---|---|---|---|---|
|  | 5 | 4 | 3 | × |   |
|  |   |   |   |   | 3 |
|  |   |   |   |   | 6 |
|  |   |   |   |   |   |
|  |   |   |   |   |   |

| Demonstration Problems  | Practice Problems |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
|---|-------------------|---|---|---|---|---|-------|---|-------|--|--|--|---|---|---|---|---|--|---|-------|-------|--|--|--|--|--|--|--|---|--|---|---|---|---|--|---|---|-------|--|--|--|--|--|--|--|---|--|--|--|-------|--|--|--|--|--|--|--|
| <p>1. (a) <math>342 \times 2 =</math></p> <div style="text-align: right;"> <table border="1" style="border-collapse: collapse; margin-left: auto;"> <tr><td>3</td><td>4</td><td>2</td></tr> <tr><td>×</td><td></td><td>2</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td></td><td></td><td></td></tr> </table> </div>   | 3                 | 4 | 2 | × |   | 2 | <hr/> |   |       |  |  |  | <p>1. (b) <math>134 \times 2 =</math></p> <div style="text-align: right;"> <table border="1" style="border-collapse: collapse; margin-left: auto;"> <tr><td>1</td><td>3</td><td>4</td></tr> <tr><td>×</td><td></td><td>2</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td></td><td></td><td></td></tr> </table> </div> | 1 | 3 | 4 | × |  | 2 | <hr/> |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| 3   | 4                 | 2 |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| ×   |                   | 2 |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| <hr/>   |                   |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
|   |                   |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| 1   | 3                 | 4 |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| ×   |                   | 2 |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| <hr/>   |                   |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
|   |                   |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| <p>2. (a) <math>145 \times 21 =</math></p> <div style="text-align: right;"> <table border="1" style="border-collapse: collapse; margin-left: auto;"> <tr><td></td><td>1</td><td>4</td><td>5</td></tr> <tr><td>×</td><td></td><td>2</td><td>1</td></tr> <tr><td colspan="4"><hr/></td></tr> <tr><td></td><td></td><td></td><td></td></tr> <tr><td>+</td><td></td><td></td><td></td></tr> <tr><td colspan="4"><hr/></td></tr> <tr><td></td><td></td><td></td><td></td></tr> </table> </div> |                   | 1 | 4 | 5 | × |   | 2     | 1 | <hr/> |  |  |  |   |   |   |   | + |  |   |       | <hr/> |  |  |  |  |  |  |  | <p>2. (b) <math>215 \times 23 =</math></p> <div style="text-align: right;"> <table border="1" style="border-collapse: collapse; margin-left: auto;"> <tr><td></td><td>2</td><td>1</td><td>5</td></tr> <tr><td>×</td><td></td><td>2</td><td>3</td></tr> <tr><td colspan="4"><hr/></td></tr> <tr><td></td><td></td><td></td><td></td></tr> <tr><td>+</td><td></td><td></td><td></td></tr> <tr><td colspan="4"><hr/></td></tr> <tr><td></td><td></td><td></td><td></td></tr> </table> </div> |  | 2 | 1 | 5 | × |  | 2 | 3 | <hr/> |  |  |  |  |  |  |  | + |  |  |  | <hr/> |  |  |  |  |  |  |  |
|   | 1                 | 4 | 5 |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| ×   |                   | 2 | 1 |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| <hr/>   |                   |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
|   |                   |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| +   |                   |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| <hr/>   |                   |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
|   |                   |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
|   | 2                 | 1 | 5 |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| ×   |                   | 2 | 3 |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| <hr/>   |                   |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
|   |                   |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| +   |                   |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| <hr/>   |                   |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
|   |                   |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |
| <p>Answers: 1. (b) 268; 2. (b) 4,945</p>  |                   |   |   |   |   |   |       |   |       |  |  |  |   |   |   |   |   |  |   |       |       |  |  |  |  |  |  |  |   |  |   |   |   |   |  |   |   |       |  |  |  |  |  |  |  |   |  |  |  |       |  |  |  |  |  |  |  |

**Consider  $423 \times 10$**

Notice that:

|       |   |   |   |
|-------|---|---|---|
|       | 4 | 2 | 3 |
| ×     |   | 1 | 0 |
| <hr/> |   |   |   |
|       |   |   |   |
| +     |   |   |   |
| <hr/> |   |   |   |
|       |   |   |   |

We can use a shortcut to achieve the same result

Shortcut:

$$423 \times 10 = 4,230$$

Similarly:

$$423 \times 100 = 42,300$$

And:

$$423 \times 1,000 = 423,000$$

| Demonstration Problems                  | Practice Problems           |
|---|-----------------------------|
| 3. (a) $2,356 \times 10 =$              | 3. (b) $2,356 \times 100 =$ |
| 4. (a) $127 \times 10,000 =$            | 4. (b) $145 \times 1,000 =$ |
| Answers: 3. (b) 235,600; 4. (b) 145,000 |                             |

### Finding the Area of a Rectangle

To find the amount of fertilizer needed for a lawn, the amount of paint needed to paint a wall, or the amount of carpet needed to cover a room, we must find the area of the given region. If the region is a rectangle, we use the formula

Area of a rectangle = length  $\times$  width

To find the area of the following rectangles, multiply the length of each one by the width.

3 inches



5 inches

34 feet



125 feet