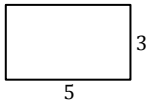
2.1 The Language of Algebra

Formula for the Area of a Rectangle

Recall that we learned in section 1.4 that the area of a rectangle is equal to the length of its base times the length of its height. Consider the following rectangles and their areas:

Rectangle	Area = base × height
	$2 \times 1 = 2$ square units
	$3 \times 2 = 6$ square units
	$5 \times 3 = 15$ square units
	$b \times h = bh$ square units $A = bh$

Rectangles come in many different sizes and shapes, but the procedure for finding the area of any rectangle remains the same. Notice that in the last example, the letters (also known as variables), A, b, and h, are convenient and meaningful placeholders for the values of *area*, *base*, and *height* for any given rectangle. The letters allow us to define a formula for the area of any rectangle as

$$A = bh$$

Variable: A symbol that represents a quantity in a mathematical expression, whose value may vary.

Using Exponential Notation

Consider the expression $5 \cdot 5 \cdot 5$

We can represent this product with the following shorthand notation:

exponent

$5^3 = 5 \cdot 5 \cdot 5$

base

The *exponent*, 3, indicates how many times the *base*, 5, is used as a factor.

We say 5^3 is written in *exponential notation*.

Exponential Notation	**Read as…**
7^2	7 squared
5^3	5 cubed
9^4	9 to the 4^{th} power
12^5	12 to the 5^{th} power

Demonstration Problems	*Practice Problems*
Write using exponential notation.	Write using exponential notation.
1. (a) $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$	**1. (b)** $7 \cdot 7 \cdot 7 \cdot 7$
2. (a) $x \cdot x \cdot x \cdot x$	**2. (b)** $m \cdot m \cdot m \cdot m \cdot m \cdot m$
Evaluate **3. (a)** 12^2	Evaluate **3. (b)** 11^2
4. (a) 2^5	**4. (b)** 3^4
	Answers: **1. (b)** 7^4; **2. (b)** m^6; **3. (b)** 121; **4. (b)** 81

Order of Operations

Parentheses, *brackets*, and *fraction bars* are grouping symbols which are used to define which operations in a mathematical expression are to be done first.

Demonstration Problems	*Practice Problems*
Simplify **5. (a)** $8 + (3 \cdot 5)$	Simplify **5. (b)** $5 + (2 \cdot 4)$
6. (a) $[8 + (3 - 2)] \cdot 5$	**6. (b)** $4 \cdot [5 + (3 - 1)]$
7. (a) $\dfrac{10 - 2}{4}$	**7. (b)** $\dfrac{10}{3 - 1}$
	Answers: **5. (b)** 13; **6. (b)** 28; **7. (b)** 5

When there are no parentheses, which operations are performed first?
Let's explore the following mathematical expressions:

$56 + 3 \cdot 2$	$4 + 6 \cdot 32$
If we add first, we get: $(56 + 3) \cdot 2 =$	If we add first, we get: $(4 + 6) \cdot 32 =$
If we multiply first, we get: $56 + (3 \cdot 2) =$	If we multiply first, we get: $4 + (6 \cdot 32) =$
Which was easier to calculate?	Which was easier to calculate?

From this exploration, we can see that we get different answers when we perform mathematical operations in different orders. To avoid the temptation of choosing the order that is easiest to follow, mathematicians have decided upon an order of operations.

Order of Operations

1st Simplify the expression in the inner most grouping symbol.
2nd Evaluate exponential expressions.
3rd Perform multiplication and division in order from left to right.
4th Perform addition and subtraction in order from left to right.

Demonstration Problems	*Practice Problems*
Simplify the following expressions: **8. (a)** $5^2 + 3(7 - 5)$	Simplify the following expressions: **8. (b)** $4^3 - 4(2 + 3)$
9. (a) $20 \div 5 \times 2$	**9. (b)** $30 \div 6 \times 3$
10. (a) $5 + 2^3 + 3[6 - 3(4 - 2)]$	**10. (b)** $9 + 5^3 - [4(9 + 3)]$
11. (a) $3(1 + 9 \bullet 6) - 4^2$	**11. (b)** $5(2 + 8 \bullet 4) - 7^2$
	Answers: **8. (b)** 44; **9. (b)** 15; **10. (b)** 86; **11. (b)** 121

Algebraic Expressions

An *algebraic expression* is a combination of numbers and variables built up with addition, subtraction, multiplication, division, or exponentiation.

Suppose we have two real numbers a and b, $b \neq 0$. Then

The sum of a and b	is	$a + b$
The difference of a and b	is	$a - b$
The product of a and b	is	ab or $a \cdot b$
The quotient of a and b	is	$\frac{a}{b}$ or $b\overline{)a}$ or $a \div b$
The square of a	is	a^2
b more than a	is	$a + b$
b less than a	is	$a - b$

Demonstration Problems	*Practice Problems*
Translate each verbal expression into an algebraic one.	Translate each verbal expression into an algebraic one.
12. (a) The square of a number	**12. (b)** The quotient of a number and 4
13. (a) The difference of 3 and a number	**13. (b)** The square of twice a number
14. (a) The sum of 3 and a number	**14. (b)** Four more than three times a number
15. (a) Three less than twice a number	**15. (b)** The product of 2 and a number
Answers: **12. (b)** $\frac{x}{4}$; **13. (b)** $(2x)^2$; **14. (b)** $3x + 4$; **15. (b)** $2x$	