
2.2 Evaluate, Simplify, and Translate Expressions

Algebraic Expressions, Terms, and Coefficients

An *algebraic expression* is a combination of numbers and variables built up with addition, subtraction, multiplication, division, or exponentiation. Each algebraic expression is a term or a sum of terms where a *term* is a number, a variable, or a product of numbers and variables. The numerical factor of each term is called the *coefficient* of the term. *Like terms* are terms that have the same variables raised to the same exponents.

Algebraic Expression
3
x
$7x + 5y$
$7a^2$
$3x^2 + 8x - 12$

Terms	Coefficients of Terms
3	3
x	1 (because $x = 1 \cdot x$)
$5y$	5
$7a^2$	7
$12x^2yz$	12

Like Terms	Unlike Terms
3, 9	3, x
x , $8x$	x , $8y$
$7a^2$, $3a^2$	$7a$, $3a^2$
$12x^2y$, $2x^2y$	$12x^2y$, $2xy^2$

Translation between Words and Algebraic Expressions

Suppose we have two real numbers a and b , $b \neq 0$. Then

The sum of a and b	is	$a + b$
The difference of a and b	is	$a - b$
The product of a and b	is	ab or $a \cdot b$
The quotient of a and b	is	$\frac{a}{b}$ or $b \overline{)a}$ or $a \div b$
The square of a	is	a^2
b more than a	is	$a + b$
b less than a	is	$a - b$



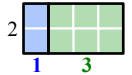
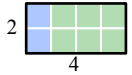
<i>Demonstration Problems</i>	<i>Practice Problems</i>
<p>Translate each verbal expression into an algebraic one.</p> <p>1. (a) The square of a number</p> <p>2. (a) The difference of 3 and a number</p> <p>3. (a) The sum of 3 and a number</p> <p>4. (a) Three less than twice a number</p>	<p>Translate each verbal expression into an algebraic one.</p> <p>1. (b) The quotient of a number and 4</p> <p>2. (b) The square of twice a number</p> <p>3. (b) Four more than three times a number</p> <p>4. (b) The product of 2 and a number</p>
<p>Translate each algebraic expressions into words.</p> <p>5. (a) $x + 12$</p> <p>The _____ of a number and 12.</p> <p>6. (a) $12x$</p> <p>The _____ of 12 and a number.</p> <p>7. (a) $5x + 12$</p> <p>12 _____ five times a number.</p> <p>8. (a) $2x - 12$</p> <p>12 _____ twice a number.</p>	<p>Translate each algebraic expressions into words.</p> <p>5. (b) $x - 12$</p> <p>The _____ of a number and 12.</p> <p>6. (b) $2x$</p> <p>_____ a number.</p> <p>7. (b) $3x + 7$</p> <p>7 _____ three times a number.</p> <p>8. (b) $2x + 10$</p> <p>The _____ of twice a number and 10.</p>
<p>Answers: 1. (b) $\frac{x}{4}$; 2. (b) $(2x)^2$; 3. (b) $3x + 4$; 4. (b) $2x$; 5. (b) difference; 6. (b) twice; 7. (b) more than; 8. (b) sum;</p>	

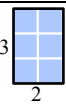
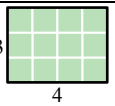
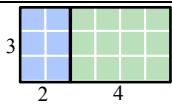
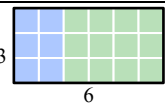
Distributive Property



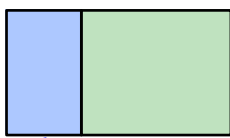

Recall in section 1.4, that for any real numbers a , b , and c


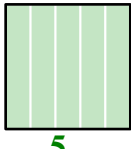
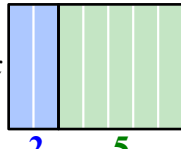
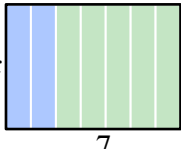
$$a(b + c) = a \cdot b + a \cdot c$$

The following areas of rectangles give a visual representation of the distributive property:

Demonstration that $2(1 + 3) = 2 \times 1 + 2 \times 3$				
	Separate		Pushed together	Merged into one rectangle
Rectangles				
Areas	$2 \times 1 = 2$	$2 \times 3 = 6$	$2 \times 1 + 2 \times 3 = 2 + 6 = 8$	$2 \times (1 + 3) = 2 \times 4 = 8$

Demonstration that $3(2 + 4) = 3 \times 2 + 3 \times 4$				
	Separate		Pushed together	Merged into one rectangle
Rectangles				
Areas	$3 \times 2 = 6$	$3 \times 4 = 12$	$3 \times 2 + 3 \times 4 = 6 + 12 = 18$	$3 \times (2 + 4) = 3 \times 6 = 18$

Demonstration that $a(b + c) = ab + ac$				
	Separate		Pushed together	Merged into one rectangle
Rectangles				
Areas	ab	ac	$ab + ac$	$a(b + c)$

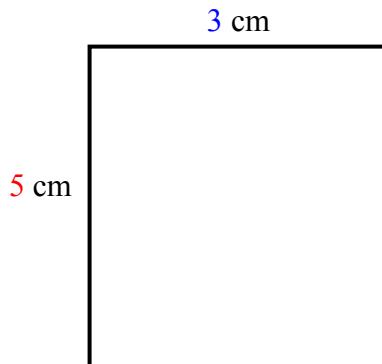
Demonstration that $2x + 5x = (2 + 5)x = 7x$				
	Separate		Pushed together	Merged into one rectangle
Rectangles				
Areas	$2x$	$5x$	$2x + 5x$	$(2 + 5)x = 7x$

We can use the distributive property to simplify algebraic expressions that have like terms (terms that have the same variables raised to the same exponents).

<i>Demonstration Problems</i>	<i>Practice Problems</i>
Simplify 9. (a) $3x + 7x$	Simplify 9. (b) $2x + 12x$
10. (a) $5xy + 4xy$	10. (b) $7xy + xy$
11. (a) $2m + m$	11. (a) $4m + m$
12. (a) $9x + 5 + 3 + 5x$	12. (a) $3x + 1 + 7 + 9x$
Answers: 9. (b) $14x$; 10. (b) $8xy$; 11. (b) $5m$; 12. (b) $12x + 8$;	

Evaluating Algebraic Expressions

Consider the rectangle shown below:



We can find the perimeter of this rectangle by evaluating the formula

$$P = 2L + 2W$$

with $L = 5$ and $W = 3$.

This gives us

$$\begin{aligned} 2 \times 5 + 2 \times 3 \\ = 10 + 6 \\ = 16 \end{aligned}$$

So then the perimeter is 16 cm.

To evaluate an algebraic expression in general, we substitute given values for the variables used in the expression.

<i>Demonstration Problems</i>	<i>Practice Problems</i>
Evaluate each expression using $a = 2$, $b = 5$, and $c = 1$ 13. (a) $b - a$ 14. (a) $b^2 - 4ac$	Evaluate each expression using $a = 1$, $b = 2$, and $c = 6$ 13. (b) $c - a$ 14. (b) $c^2 - 4ab$
Answers: 16. (b) 5; 17. (b) 28	