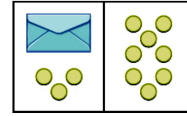


### 2.3 Solving Equations Using Addition and Subtraction Properties of Equality

Let's play a game of "How many counters are in the envelope?"



Here are the rules:

1. The same number of counters are placed on the left side and right side of a divided board.
2. Some of the counters are hidden in an envelope on the left side of the board.

We will compare this game to solving an algebraic equation simultaneously.

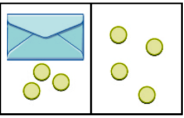
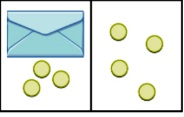
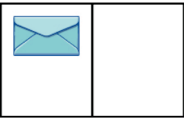
Game 1:

Counters and Envelope		Algebraic Equation	
How many counters are in the envelope?		$x + 3 = 8$	What value of $x$ will make this a true statement?
Remove 3 counters from both sides of the board.		$x + 3 - 3 = 8 - 3$	Subtract 3 from both sides of the equation.
There are 5 counters in the envelope.		$x = 5$	5 is the solution of the equation $x + 3 = 8$

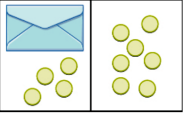
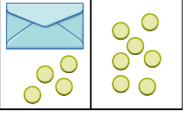

Game 2:

Counters and Envelope		Algebraic Equation	
How many counters are in the envelope?		$x + 4 = 5$	What value of $x$ will make this a true statement?
Remove 4 counters from both sides of the board.		$x + 4 - 4 = 5 - 4$	Subtract 4 from both sides of the equation.
There is 1 counter in the envelope.		$x = 1$	1 is the solution of the equation $x + 4 = 5$

Game 3 (Parts of this game are left for you to complete):

Counters and Envelope		Algebraic Equation	
How many counters are in the envelope?		$x + 3 = 4$	What value of $x$ will make this a true statement?
Remove _____ counters from both sides of the board.		$x + 3 - \underline{\quad} = 4 - \underline{\quad}$	Subtract _____ from both sides of the equation.
There are/is _____ counter(s) in the envelope.		$x = \underline{\quad}$	_____ is the solution of the equation $x + 3 = 4$

Game 4 (Parts of this game are left for you to complete):

Counters and Envelope		Algebraic Equation	
How many counters are in the envelope?			What value of $x$ will make this a true statement?
Remove _____ counters from both sides of the board.			Subtract _____ from both sides of the equation.
There are _____ counters in the envelope.			_____ is the solution of the equation

Most equations would not be easily modeled using envelopes and counters, so then we have the following two properties that give us a general method for solving some equations:

#### Subtraction Property of Equality

For any real number  $c$ , if  $a = b$ , then

$$a - c = b - c$$

#### Addition Property of Equality

For any real number  $c$ , if  $a = b$ , then

$$a + c = b + c$$

**Example (a) Solve**

$x + 3 = 7$

By the **subtraction property of equality**,

$$\begin{aligned} x + 3 &= 7 \\ x + 3 - \mathbf{3} &= 7 - \mathbf{3} \\ x + 0 &= 4 \\ x &= 4 \end{aligned}$$

Alternatively, we can use the following notation:

$$\begin{array}{r} x + 3 = 7 \\ \underline{-3 \quad -3} \\ x + 0 = 4 \\ x = 4 \end{array}$$

To check the solution, we replace  $x$  with  $4$  in the original equation.

$$\begin{aligned} x + 3 &= 7 \\ \mathbf{4} + 3 &\stackrel{?}{=} 7 \\ 7 &= 7 \quad \checkmark \end{aligned}$$

The solution set notation for

$x + 3 = 7$  is  $\{4\}$ .

**Example (b) Solve**

$x - 5 = 8$

By the addition property of equality,

$$\begin{aligned} x - 5 &= 8 \\ x - 5 + \mathbf{5} &= 8 + \mathbf{5} \\ x + 0 &= 13 \\ x &= 13 \end{aligned}$$

Alternatively, we can use the following notation:

$$\begin{array}{r} x - 5 = 8 \\ \underline{\quad \mathbf{5} \quad \mathbf{5}} \\ x + 0 = 13 \\ x = 13 \end{array}$$

To check the solution, we replace  $x$  with  $13$  in the original equation.

$$\begin{aligned} x - 5 &= 8 \\ \mathbf{13} - 5 &\stackrel{?}{=} 8 \\ 8 &= 8 \quad \checkmark \end{aligned}$$

The solution set is  $\{13\}$ .

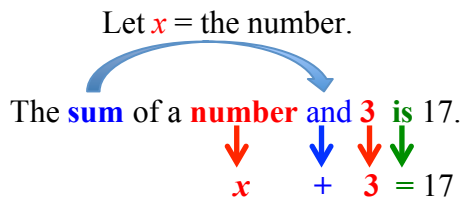
<i>Demonstration Problems</i>	<i>Practice Problems</i>
Solve each equation and check your answer. <b>1. (a)</b> $w + 43 = 85$ Check: $w + 43 = 85$	Solve each equation and check your answer. <b>1. (b)</b> $w + 38 = 54$ Check: $w + 38 = 54$
<b>2. (a)</b> $x - 4 = 34$ Check: $x - 4 = 34$	<b>2. (b)</b> $x - 12 = 42$ Check: $x - 12 = 42$
Answers: <b>1. (b)</b> $w = 16$ ; <b>2. (b)</b> $x = 54$ ;	

<i>Demonstration Problems</i>	<i>Practice Problems</i>
Solve each equation and check your answer. <b>3. (a)</b> $7 + m = 15$ Check: $7 + m = 15$	Solve each equation and check your answer. <b>3. (b)</b> $13 + m = 20$ Check: $13 + m = 20$
<b>4. (a)</b> $x - 49 = 59$ Check: $x - 49 = 59$	<b>4. (b)</b> $x - 83 = 12$ Check: $x - 83 = 12$
<b>5. (a)</b> $35 = 7 + y$ Check: $35 = 7 + y$	<b>5. (b)</b> $15 = 3 + y$ Check: $15 = 3 + y$
<b>6. (a)</b> $17 = x - 41$ Check: $17 = x - 41$	<b>6. (b)</b> $25 = x - 45$ Check: $25 = x - 45$
Answers: <b>3. (b)</b> $m = 7$ ; <b>4. (b)</b> $x = 95$ ; <b>5. (b)</b> $y = 12$ ; <b>6. (b)</b> $x = 70$	

**Word Problems**

<p><b>Strategy for Solving Word Problems</b></p> <ol style="list-style-type: none"> <li>1. Read the problem for understanding.</li> <li>2. If possible, draw a diagram to illustrate the problem.</li> <li>3. Define a variable based on the question the problem is asking.</li> <li>4. Using key words from the problem, write an equation using the details of the problem.</li> <li>5. Solve the equation.</li> <li>6. Check your answer.</li> <li>7. Answer the original question in a complete sentence.</li> </ol>	<b>Problem Solving Key Words</b>	
	<b>Addition</b>	<b>Subtraction</b>
	<ul style="list-style-type: none"> <li>• sum</li> <li>• plus</li> <li>• total</li> <li>• increased</li> <li>• more than</li> </ul>	<ul style="list-style-type: none"> <li>• combined</li> <li>• in all</li> <li>• altogether</li> <li>• perimeter</li> </ul>
	<b>Multiplication</b>	<b>Division</b>
	<ul style="list-style-type: none"> <li>• product</li> <li>• of</li> <li>• multiplied</li> <li>• times</li> </ul>	<ul style="list-style-type: none"> <li>• twice</li> <li>• by</li> <li>• factor</li> <li>• area</li> </ul>
<b>Equals</b>		
<ul style="list-style-type: none"> <li>• is</li> <li>• result</li> </ul>	<ul style="list-style-type: none"> <li>• difference</li> <li>• minus</li> <li>• decreased</li> <li>• less than</li> <li>• loss</li> </ul>	
	<ul style="list-style-type: none"> <li>• remaining</li> <li>• dropped</li> <li>• changed</li> </ul>	
	<ul style="list-style-type: none"> <li>• quotient</li> <li>• divide</li> <li>• into</li> <li>• equal pieces</li> <li>• ratio</li> </ul>	
	<ul style="list-style-type: none"> <li>• out of</li> <li>• shared</li> <li>• each</li> <li>• per</li> </ul>	
	<ul style="list-style-type: none"> <li>• same as</li> <li>• equivalent to</li> </ul>	

**Example (g)** The sum of a number and 3 is 17. What is the number?



$$\begin{aligned}
 x + 3 &= 17 \\
 \underline{-3} \quad \underline{-3} \\
 x + 0 &= 14 \\
 x &= 14
 \end{aligned}$$

Check:  $x + 3 = 17$   
 $14 + 3 = 17$

The number is 14.

<i>Demonstration Problems</i>	<i>Practice Problems</i>
<p><b>7. (a)</b> The sum of a number and 15 is 125. What is the number?</p>	<p><b>7. (b)</b> The sum of 57 and a number is 68. What is the number?</p>
<p><b>8. (a)</b> Five less than a number is 20. What is the number?</p>	<p><b>8. (b)</b> Twelve less than a number is 20. What is the number?</p>
<p>Answers: <b>7. (b)</b> The number is 11.; <b>8. (b)</b> The number is 32.</p>	