2.4 Multiples and Factors of Whole Numbers

A whole number is considered a ***multiple*** if it is a product of two whole numbers. For example, 6 is a multiple because $6 = 2 \times 3$. We say that 2 and 3 are ***factors*** of 6.

A multiplication table is convenient tool for finding patterns in multiples.

Multiplication Table

×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Notice the following patterns:

⇐ Multiples of 2 end in 0, 2, 4, 6, or 8.

⇐ Multiples of 3 have digits whose sums are multiples of 3

⇐ Multiples of 5 end in 5 or 0.

⇐ Multiples of 6 end in 0, 2, 4, 6, or 8 and have digits whose sums are multiples of 3.

⇐ Multiples of 9 have digits whose sums are multiples of 9

⇐ Multiples of 10 end in 0.

A number is considered ***divisible*** by another if the quotient of the two numbers is a whole number. Suppose we need to know the following:

Is 485 divisible by 5?
Is 1,258 divisible by 3?
Is 94 divisible by 2?
Is 792 divisible by 6?

Because of the patterns identified in the multiplication table above, we can determine if any number is divisible by 2, 3, 5, 6, 9, or 10.

Divisibility Tests	
A number is divisible by…	if…
2	the number ends in 0, 2, 4, 6, or 8
3	the sum of its digits is a multiple of 3
5	the number ends in 0 or 5
6	the number divides by 2 and 3
9	the sum of its digits is a multiple of 9
10	the number ends in 0

Demonstration Problems	*Practice Problems*
Use the divisibility tests to answer the following:	Use the divisibility tests to answer the following:
1. (a) Is 485 divisible by 5?	**1. (b)** Is 635 divisible by 5?
2. (a) Is 485 divisible by 2?	**2. (b)** Is 635 divisible by 2?
3. (a) Is 480 divisible by 6?	**3. (b)** Is 286 divisible by 6?
4. (a) Is 480 divisible by 9?	**4. (b)** Is 837 divisible by 9?
5. (a) Is 9,740 divisible by 10?	**5. (b)** Is 10,562 divisible by 10?
6. (a) Is 1,253 divisible by 3?	**6. (b)** Is 3,486 divisible by 3?
Answers: **1. (b)** yes; **2. (b)** no; **3. (b)** no; **4. (b)** yes; **5. (b)** no; **6. (b)** yes	

Prime, Composite, and Identity Numbers

A whole number is a ***prime*** number if it contains exactly 2 factors.
A whole number is a ***composite*** number if it contains finitely many, but more than 2 factors.
The numbers, 0 and 1, are ***identity*** numbers (neither prime nor composite).

A famous mathematician from antiquity, Eratosthenes developed a method to find prime numbers. This method is called the Sieve of Eratosthenes. Using the table below:

1. In the 100-table below, cross off the number 1, as it is not prime.
2. Circle the number 2, as it is prime, and then cross off all other multiples of 2.
3. Circle the number 3, and cross off all other multiples of 3.
4. Circle the number 5, and cross off all other multiples of 5.
5. Continue the process, circling the next smallest uncrossed number and crossing off each subsequent set of multiples until no more multiples can be crossed off.

The remaining numbers in the table that were not crossed off are the prime numbers that are less than 100.

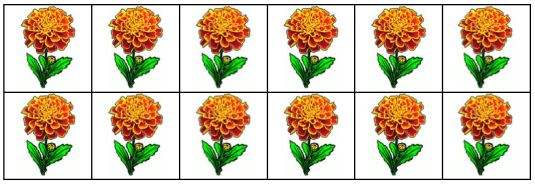
The Sieve of Eratosthenes									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Demonstration Problems	*Practice Problems*
7. (a) Is 31 a prime or composite number?	**7. (b)** Is 83 a prime or composite number?
8. (a) Is 51 a prime or composite number?	**8. (b)** Is 97 a prime or composite number?
9. (a) Is 0 a prime or composite number?	**9. (b)** Is 1 a prime or composite number?
10. (a) Is 1281 a prime or composite number?	**10. (b)** Is 1593 a prime or composite number?
11. (a) Is 127 a prime or composite number?	**11. (b)** Is 143 a prime or composite number?
Answers: **7. (b)** prime; **8. (b)** prime; **9. (b)** neither; **10. (b)** composite; **11. (b)** composite	

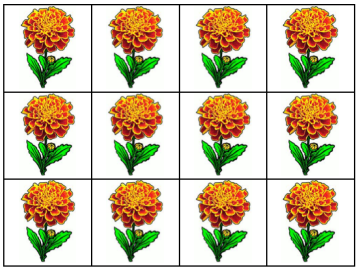
Finding all the Factors of a Number

Suppose we have 12 marigold plants and we are considering how many ways we can plant them in the shape of a rectangle.

$12 = 1 \times 12$



$12 = 2 \times 6$



$12 = 3 \times 4$

From these arrangements, we can see that 12 can be written as a product of the factors 1 and 12, 2 and 6, and 3 and 4. In order from smallest to largest, the factors of 12 are 1, 2, 3, 4, 6, and 12.

Demonstration Problems	*Practice Problems*
12. (a) Find all the factors of the number 72	**12. (b)** Find all the factors of the number 24
13. (a) Find all the factors of the number 100	**13. (b)** Find all the factors of the number 30
	Answers: **12. (b)** 1, 2, 3, 4, 6, 8, 12, 24; **13. (b)** 1, 2, 3, 5, 6, 10, 15, 30