2.5 Prime Factorization and Least Common Multiples

In section 2.4, we learned that

> A whole number is considered a ***multiple*** if it is a product of two whole numbers. For example, 6 is a multiple because $6 = 2 \times 3$. We say that 2 and 3 are ***factors*** of 6.

We can also say that 2×3 is a ***factorization*** of 6. Here are some other factorizations:

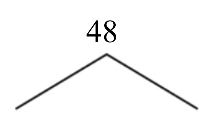
Number	Factorization	Different Factorization	
15	1×15	3×5	⇐ prime factorization of 15
18	3×6	$2 \times 3 \times 3$	⇐ prime factorization of 18
40	4×10	5×8	
72	8×9	$2 \times 4 \times 9$	

We can see that many numbers have a factorization that is a product of more than two numbers. When a factorization is a product of prime numbers only, we say this is a ***prime factorization***. For example, 3×5 is a prime factorization of 15 and $2 \times 3 \times 3$ is a prime factorization of 18.

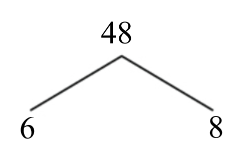
Factor Tree Method for Finding Prime Factorizations

To find the prime factorization of larger numbers such as 48, we can use the Factor Tree Method.

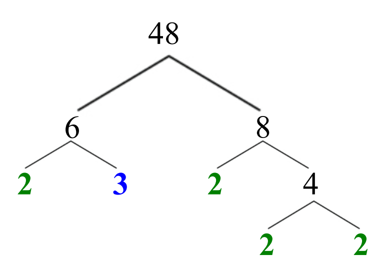
Step 1: Draw two lines that "branch" below the number 48.



Step 2: Find any two numbers whose product is 48. Place those numbers at the ends of the two branches.



Step 3: Repeat the process for each of the factors until the new factors are prime number factors.



Step 4: Write the number 48 as a product of its prime factors.

$$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 2^4 \cdot 3$$

Demonstration Problems	*Practice Problems*
Use a factor tree to find the prime factorization of each of the following:	Use a factor tree to find the prime factorization of each of the following:
1. (a) 30	**1. (b)** 42
2. (a) 100	**2. (b)** 200
3. (a) 630	**3. (b)** 1650
	Answers: **1. (b)** $2 \times 3 \times 7$; **2. (b)** $2 \times 2 \times 2 \times 5 \times 5$; **3. (b)** $2 \times 3 \times 5 \times 5 \times 11$

In section 2.4, we performed the sieve of Eratosthenes with the following results:

The Sieve of Eratosthenes									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The unshaded numbers are all the prime numbers that are smaller than 100. This chart can be very useful in determining the prime factorization of large numbers. We will use this chart along with the Ladder Method for finding the prime factorization of numbers.

Ladder Method for Finding Prime Factorizations

Find the prime factorization of 48.

Step 1: Divide 48 by the smallest prime that divides 48.	**Step 2:** Divide **24** by the smallest prime that divides 24.	**Step 3:** Divide **12** by the smallest prime that divides 12.	**Step 4:** Divide **6** by the smallest prime that divides 6.	**Step 5:** Since 3 is prime. The dividing is done.
2 | 48 **24**	2 | 48 2 | 24 **12**	2 | 48 2 | 24 2 | 12 **6**	2 | 48 2 | 24 2 | 12 2 | 6 3	

$$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$$

Demonstration Problems	*Practice Problems*
Find the prime factorization of each of the following:	Find the prime factorization of each of the following:
4. (a) 75	**4. (b)** 98
5. (a) 490	**5. (b)** 105
6. (a) 990	**6. (b)** 980

Answers: **4. (b)** $2 \times 7 \times 7$; **5. (b)** $3 \times 5 \times 7$; **6. (b)** $2 \times 2 \times 5 \times 7 \times 7$

Least Common Multiple

Consider the multiples of 5 and 4 as shown below:

5	5 • 1 = 5	5 • 2 = 10	5 • 3 = 15	5 • 4 = 20	5 • 5 = 25	5 • 6 = 30	5 • 7 = 35	5 • 8 = 40	5 • 9 = 45	5 • 10 = 50
4	4 • 1 = 4	4 • 2 = 8	4 • 3 = 12	4 • 4 = 16	4 • 5 = 20	4 • 6 = 324	4 • 7 = 28	4 • 8 = 32	4 • 9 = 36	4 • 10 = 40

Common multiples of 4 and 5 are 20, 40, 60, 80, ...
The least common multiple of 4 and 5 is 20.

The smallest number in the list of common multiples is the *least common multiple*.
We can denote the least common multiple of 4 and 5 as **LCM (4, 5) = 20**.

Demonstration Problems	*Practice Problems*
7. (a) LCM (20, 25) =	**7. (b)** LCM (10, 25) =
8. (a) LCM (12, 18) =	**8. (b)** LCM (15, 20) =
9. (a) LCM (60, 70) =	**9. (b)** LCM (70, 84) =

Answers: **7. (b)** 50; **8. (b)** 60; **9. (b)** 420

Using the Ladder Method to Find the Least Common Multiple

LCM (60, 80) =

Step 1: Divide 60 and 80 by the smallest prime that divides them both.	**Step 2:** Divide 30 and 40 by the smallest prime that divides them both.	**Step 3:** Divide 15 and 20 by the smallest prime that divides them both.	**Step 4:** Since no prime divides both 3 and 4, the dividing is done.
$\begin{array}{r\|l} 2 & 60 \quad 80 \\ \hline & 30 \quad 40 \end{array}$	$\begin{array}{r\|l} 2 & 60 \quad 80 \\ \hline 2 & 30 \quad 40 \\ \hline & 15 \quad 20 \end{array}$	$\begin{array}{r\|l} 2 & 60 \quad 80 \\ \hline 2 & 30 \quad 40 \\ \hline 5 & 15 \quad 20 \\ \hline & 3 \quad 4 \end{array}$	

LCM(60, 80) = $2 \times 2 \times 5 \times 3 \times 4 = 240$

Demonstration Problems	*Practice Problems*
10. (a) LCM(24, 30)	**10. (b)** LCM(32, 40)
	Answer: **10. (b)** 160