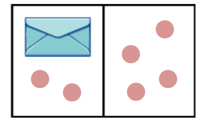
3.5 Solving Equations Using Integers; The Division Property of Equality



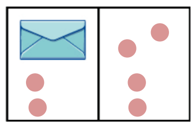
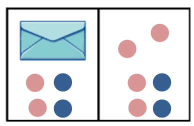
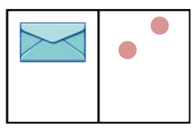
Let's return to our game of "How many counters are in the envelope?"

We use the same rules, but our counters can be blue or red:

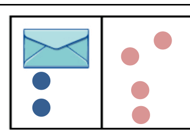
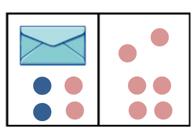
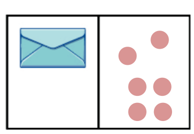
1. The same value of integer counters are placed on the left side and right side of a divided board.
2. Some of the counters are hidden in an envelope on the left side of the board.

We will compare this game to solving an algebraic equation simultaneously.

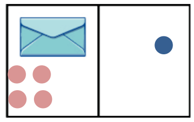
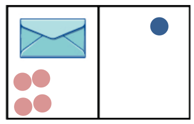
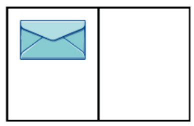
Game 1:

Counters and Envelope		Algebraic Equation	
How many counters are in the envelope?		$x + (-2) = -4$ or $x - 2 = -4$	What value of x will make this a true statement?
Add 2 blue counters to both sides of the board, then remove any "0" pairs.		$x - 2 + 2 = -4 + 2$	Add 2 to both sides of the equation.
There are 2 red counters in the envelope.		$x = -2$	-2 is the solution of the equation $x - 2 = -4$

Game 2:

Counters and Envelope		Algebraic Equation	
How many counters are in the envelope?		$x + 2 = -4$	What value of x will make this a true statement?
Add 2 red counters to both sides of the board, then remove any "0" pairs.		$x+2+(-2)=-4+(-2)$	Add -2 to both sides of the equation.
There are 6 red counters in the envelope.		$x = -6$	-6 is the solution of the equation $x + 2 = -4$

Game 3 (Parts of this game are left for you to complete):

Counters and Envelope		**Algebraic Equation**	
How many counters are in the envelope?		$x + (-4) = 1$ or $x - 4 = 1$	What value of x will make this a true statement?
Add __________ counters to both sides of the board, then remove any "0" pairs.		$x - 4 +$ ____ $= 1 +$ ____	Add__________ to both sides of the equation.
There are ________ counters in the envelope.		$x =$	_____ is the solution of the equation $x - 4 = 1$

Demonstration Problems	*Practice Problems*
Solve each equation and check your answer.	Solve each equation and check your answer.
1. (a) $w - 6 = -4$ Check: $w - 6 = -4$	**1. (b)** $w - 15 = -14$ Check: $w - 15 = -14$
2. (a) $x + 4 = -8$ Check: $x + 4 = -8$	**2. (b)** $x + 5 = -12$ Check: $x + 5 = -12$

Answers: **1. (b)** $w = 1$; **2. (b)** $x = -17$;

Demonstration Problems	*Practice Problems*
Solve each equation and check your answer.	Solve each equation and check your answer.
3. (a) $-7 + m = 8$ Check: $-7 + m = 8$	**3. (b)** $-13 + m = 20$ Check: $-13 + m = 20$
4. (a) $x - 9 = -12$ Check: $x - 9 = -12$	**4. (b)** $x - 83 = -12$ Check: $x - 83 = -12$
5. (a) $-35 = 7 + y$ Check: $-35 = 7 + y$	**5. (b)** $-15 = 3 + y$ Check: $-15 = 3 + y$
6. (a) $17 = x - 31$ Check: $17 = x - 31$	**6. (b)** $25 = x - 35$ Check: $25 = x - 35$

Answers: **3. (b)** $m = 33$; **4. (b)** $x = 71$; **5. (b)** $y = -18$; **6. (b)** $x = 60$

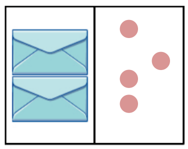
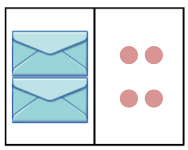
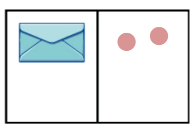
Division Property of Equality

We will now add a third rule to our game:

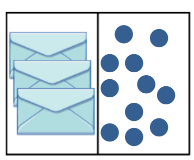
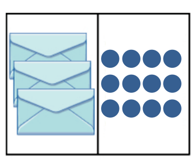
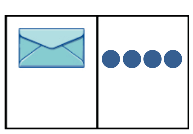
1. The same value of integer counters are placed on the left side and right side of a divided board.
2. Some of the counters are hidden in envelopes on the left side of the board.
3. Each envelope must contain the same number and same color of counters.

We will compare this game to solving an algebraic equation simultaneously.

Game 4:

Counters and Envelope		**Algebraic Equation**	
How many counters are in each envelope?		$2x = -4$	What value of x will make this a true statement?
Divide the counters on the right side into 2 groups of equal size.		$\frac{2x}{2} = \frac{-4}{2}$	Divide both sides of the equation by 2.
There are 2 red counters in each envelope.		$x = -2$	–2 is the solution of the equation $2x = -4$

Game 5:

Counters and Envelope		**Algebraic Equation**	
How many counters are in each envelope?		$3x = 12$	What value of x will make this a true statement?
Divide the counters on the right side into 3 groups of equal size.		$\frac{3x}{3} = \frac{12}{3}$	Divide both sides of the equation by 3.
There are 4 blue counters in each envelope.		$x = 4$	4 is the solution of the equation $3x = 12$

The new envelopes and counters game illustrates the following:

Division Property of Equality

If $a = b$, and $c \neq 0$, then $a \div c = b \div c$.

Demonstration Problems	*Practice Problems*
Solve each equation and check your answer.	Solve each equation and check your answer.
7. (a) $5m = 15$ Check: $5m = 15$	**7. (b)** $4m = 20$ Check: $4m = 20$
8. (a) $3x = -48$ Check: $3x = -48$	**8. (b)** $3x = -12$ Check: $3x = -12$
9. (a) $-35 = 7y$ Check: $-35 = 7y$	**9. (b)** $48 = 6x$ Check: $48 = 6x$
10. (a) $54 = 6x$ Check: $54 = 6x$	**10. (b)** $-49 = 7y$ Check: $-49 = 7y$

Answers: **7. (b)** $m = 5$; **8. (b)** $x = -4$; **9. (b)** $y = 8$; **10. (b)** $x = -7$

We can extend the division property of equality to solve equations where the coefficient of the variable is negative.

Demonstration Problems	*Practice Problems*
Solve each equation and check your answer.	Solve each equation and check your answer.
11. (a) $-5m = 15$ Check: $-5m = 15$	**11. (b)** $-4m = 20$ Check: $-4m = 20$
12. (a) $-3x = -48$ Check: $-3x = -48$	**12. (b)** $-3x = -12$ Check: $-3x = -12$
13. (a) $-35 = -7y$ Check: $-35 = -7y$	**13. (b)** $48 = -6x$ Check: $48 = -6x$
14. (a) $54 = -6x$ Check: $54 = -6x$	**14. (b)** $-49 = -7y$ Check: $-49 = -7y$

Answers: **11. (b)** $m = -5$; **12. (b)** $x = 4$; **13. (b)** $y = -8$; **14. (b)** $x = 7$