
4.2 Multiplying and Dividing Fractions

Simplifying Fractions

To simplify fractions, we use the Equivalent Fractions Property:

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c} \quad (b \neq 0 \text{ and } c \neq 0)$$

Example $\frac{10}{25} = \frac{2 \cdot \cancel{5}}{5 \cdot \cancel{5}} = \frac{2}{5}$ or $\frac{\cancel{10}^2}{\cancel{25}_5} = \frac{2}{5}$

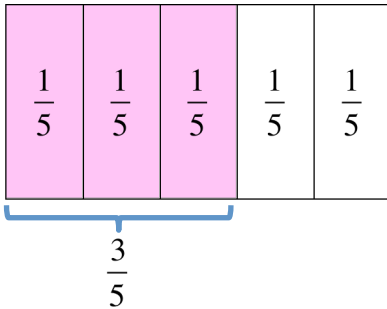
<i>Demonstration Problems</i>	<i>Practice Problems</i>
Simplify. 1. (a) $\frac{21}{35} =$	Simplify. 1. (b) $\frac{25}{55} =$
2. (a) $\frac{30}{42} =$	2. (b) $\frac{15}{45} =$
3. (a) $\frac{42}{105} =$	3. (b) $\frac{70}{90} =$
4. (a) $\frac{66}{140} =$	4. (b) $\frac{60}{126} =$
Answers: 1. (b) $\frac{5}{11}$; 2. (b) $\frac{1}{3}$; 3. (b) $\frac{7}{9}$; 4. (b) $\frac{10}{21}$	

Multiplying Fractions

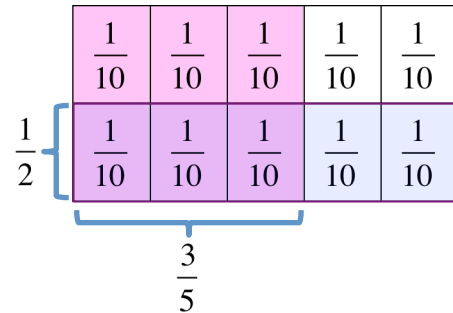
Recall that we can represent fractions using rectangular strips.

Example

We represent $\frac{3}{5}$ as follows:



We can use the same strip to represent $\frac{1}{2} \cdot \frac{3}{5}$ as follows:



This shows that $\frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$

In general, to multiply fractions, we can use the following property:

Multiplication of Fractions Property

For any real numbers a , b , c , and d , ($b \neq 0$ and $d \neq 0$)

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

Example

Multiply $\frac{2}{3} \cdot \frac{1}{2}$

Solution

$$\frac{2}{3} \cdot \frac{1}{2} = \frac{2 \cdot 1}{3 \cdot 2} = \frac{2}{6} \text{ and } \frac{2^1}{\cancel{2}_3} = \frac{1}{3}$$

Shortcut

$$\frac{\cancel{2}^1}{3} \cdot \frac{1}{\cancel{2}_1} = \frac{1}{3}$$

<i>Demonstration Problems</i>	<i>Practice Problems</i>
<p>5. (a) $\frac{2}{3} \cdot \frac{5}{7} = \underline{\hspace{2cm}}$</p>	<p>5. (b) $\frac{3}{5} \cdot \frac{4}{11} = \underline{\hspace{2cm}}$</p>
<p>6. (a) $\frac{4}{5} \cdot \frac{15}{49} = \underline{\hspace{2cm}}$</p>	<p>6. (b) $\frac{6}{7} \cdot \frac{5}{4} = \underline{\hspace{2cm}}$</p>
<p>Answers: 5. (b) $\frac{12}{55}$; 6. (b) $\frac{15}{14}$</p>	

Division of Fractions Property

For any real numbers a , b , c , and d , ($b \neq 0$, $c \neq 0$, and $d \neq 0$)

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Example	
Step	Divide $5\frac{1}{2} \div \frac{3}{4}$
1.	Write the mixed number as an improper fraction. $5\frac{1}{2} \div \frac{3}{4} = \frac{11}{2} \div \frac{3}{4}$
2.	Rewrite the division problem as an equivalent multiplication problem by applying this rule: A number <i>divided</i> by a fraction is equivalent to the number <i>multiplied</i> by the reciprocal* of the fraction. $\frac{11}{2} \div \frac{3}{4} = \frac{11}{2} \cdot \frac{4}{3}$
3.	Multiply and simplify the result. $\frac{11}{\cancel{2}_1} \cdot \frac{\cancel{4}^2}{3} = \frac{11}{1} \cdot \frac{2}{3} = \frac{22}{3}$

* A fraction is a **reciprocal** of a another fraction if the product of the two fractions is 1.

For example, $\frac{3}{4} \cdot \frac{4}{3} = \frac{3 \cdot 4}{4 \cdot 3} = \frac{12}{12} = 1$, so then $\frac{3}{4}$ is the reciprocal of $\frac{4}{3}$.

<i>Demonstration Problems</i>	<i>Practice Problems</i>
7. (a) $\frac{2}{3} \div \frac{4}{5} = \text{---} \cdot \text{---} = \text{---}$	7. (b) $\frac{4}{7} \div \frac{5}{14} = \text{---} \cdot \text{---} = \text{---}$
8. (a) $\frac{2}{5} \div \frac{2}{15} = \text{---} \cdot \text{---} = \text{---}$	8. (b) $\frac{3}{8} \div \frac{3}{4} = \text{---} \cdot \text{---} = \text{---}$
Answers: 7. (b) $\frac{8}{5}$; 8. (b) $\frac{1}{2}$	

Recall from section 2.2 that we can simplify algebraic expressions such as, $2x + 5x$ by combining like terms.

$$2x + 5x = 7x$$

We can also simplify algebraic expressions such as $\frac{2x}{5x}$ by removing any factors of 1.

$$\frac{2x}{5x} = \frac{2 \cdot \cancel{x}}{5 \cdot \cancel{x}} = \frac{2}{5}$$

Also, we can apply rules of signed numbers when simplifying algebraic expressions such

as $-\frac{3}{4x} \cdot \frac{x}{3}$.

$$-\frac{3}{4x} \cdot \frac{x}{3} = -\frac{3 \cdot x}{4x \cdot 3} = -\frac{\cancel{3} \cdot \cancel{x}}{4 \cdot \cancel{3} \cdot \cancel{x}} = -\frac{1}{4}$$

<i>Demonstration Problems</i>	<i>Practice Problems</i>
Simplify.	Simplify.
9. (a) $-\frac{3}{4} \cdot \frac{5x}{6x} =$	9. (b) $-\frac{2a}{3a} \cdot \frac{6}{5} =$
10. (a) $\frac{2}{5} \div \frac{p}{q} = \text{---} \cdot \text{---} = \text{---}$	10. (b) $\frac{3}{8} \div \frac{a}{b} = \text{---} \cdot \text{---} = \text{---}$
11. (a) $\left(-\frac{2}{5}\right) \div \left(-\frac{4}{x}\right) = \text{---} \cdot \text{---} = \text{---}$	11. (b) $\left(-\frac{3}{7}\right) \div \left(-\frac{12}{a}\right) = \text{---} \cdot \text{---} = \text{---}$
Answers: 9. (b) $-\frac{4}{5}$; 10. (b) $\frac{3b}{8a}$; 11. (b) $\frac{a}{28}$	