4.2 Multiplying and Dividing Fractions

Simplifying Fractions

To simplify fractions, we use the Equivalent Fractions Property:

$$\frac{a}{b} = \frac{a \bullet c}{b \bullet c} \qquad (b \neq 0 \text{ and } c \neq 0)$$

Example $\frac{10}{25} = \frac{2 \bullet 5}{5 \bullet 5} = \frac{2}{5}$ or $\frac{10^{2}}{25_{5}} = \frac{2}{5}$

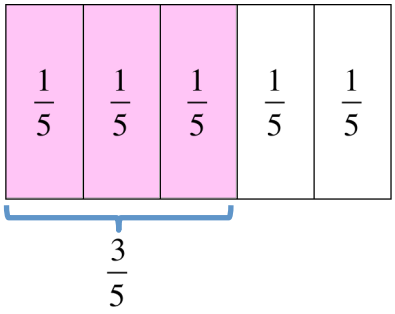
Demonstration Problems	*Practice Problems*
Simplify. **1. (a)** $\frac{21}{35} =$	Simplify. **1. (b)** $\frac{25}{55} =$
2. (a) $\frac{30}{42} =$	**2. (b)** $\frac{15}{45} =$
3. (a) $\frac{42}{105} =$	**3. (b)** $\frac{70}{90} =$
4. (a) $\frac{66}{140} =$	**4. (b)** $\frac{60}{126} =$
	Answers: **1. (b)** $\frac{5}{11}$; **2. (b)** $\frac{1}{3}$; **3. (b)** $\frac{7}{9}$; **4. (b)** $\frac{10}{21}$

Multiplying Fractions

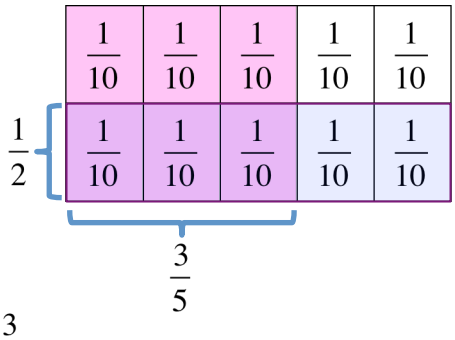
Recall that we can represent fractions using rectangular strips.

Example

We represent $\frac{3}{5}$ as follows:



We can use the same strip to represent $\frac{1}{2} \bullet \frac{3}{5}$ as follows:



This shows that $\frac{1}{2} \bullet \frac{3}{5} = \frac{3}{10}$

In general, to multiply fractions, we can use the following property:

Multiplication of Fractions Property

For any real numbers a, b, c, and d, ($b \neq 0$ and $d \neq 0$)

$$\frac{a}{b} \bullet \frac{c}{d} = \frac{a \bullet c}{b \bullet d}$$

Example Multiply $\frac{2}{3} \bullet \frac{1}{2}$

Solution

$$\frac{2}{3} \bullet \frac{1}{2} = \frac{2 \bullet 1}{3 \bullet 2} = \frac{2}{6} \text{ and } \frac{\cancel{2}^{1}}{\cancel{6}_{3}} = \frac{1}{3}$$

Shortcut

$$\frac{\cancel{2}^{1}}{3} \bullet \frac{1}{\cancel{2}_{1}} = \frac{1}{3}$$

Demonstration Problems	*Practice Problems*
5. (a) $\frac{2}{3} \bullet \frac{5}{7} =$ ———	**5. (b)** $\frac{3}{5} \bullet \frac{4}{11} =$ ———
6. (a) $\frac{4}{5} \bullet \frac{15}{49} =$ ———	**6. (b)** $\frac{6}{7} \bullet \frac{5}{4} =$ ———
	Answers: **5. (b)** $\frac{12}{55}$; **6. (b)** $\frac{15}{14}$

Division of Fractions Property

For any real numbers a, b, c, and d, ($b \neq 0$, $c \neq 0$, and $d \neq 0$)

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$$

Step	Example	
	Divide $5\frac{1}{2} \div \frac{3}{4}$	
1.	Write the mixed number as an improper fraction.	$5\frac{1}{2} \div \frac{3}{4} = \frac{11}{2} \div \frac{3}{4}$
2.	Rewrite the division problem as an equivalent multiplication problem by applying this rule: A **number** *divided* by a **fraction** is equivalent to the **number** *multiplied* by the **reciprocal* of the fraction**.	$\frac{11}{2} \div \frac{3}{4} = \frac{11}{2} \bullet \frac{4}{3}$
3.	Multiply and simplify the result.	$\frac{11}{\cancel{2}_1} \bullet \frac{\cancel{4}^2}{3} = \frac{11}{1} \bullet \frac{2}{3} = \frac{22}{3}$

* A fraction is a **reciprocal** of a another fraction if the product of the two fractions is 1.

For example, $\frac{3}{4} \bullet \frac{4}{3} = \frac{3 \bullet 4}{4 \bullet 3} = \frac{12}{12} = 1$, so then $\frac{3}{4}$ is the reciprocal of $\frac{4}{3}$.

Demonstration Problems	*Practice Problems*
7. (a) $\frac{2}{3} \div \frac{4}{5} =$ ____ $\bullet$ ____ $=$ ____	**7. (b)** $\frac{4}{7} \div \frac{5}{14} =$ ____ $\bullet$ ____ $=$ ____
8. (a) $\frac{2}{5} \div \frac{2}{15} =$ ____ $\bullet$ ____ $=$ ____	**8. (b)** $\frac{3}{8} \div \frac{3}{4} =$ ____ $\bullet$ ____ $=$ ____
	Answers: **7. (b)** $\frac{8}{5}$; **8. (b)** $\frac{1}{2}$

Recall from section 2.2 that we can simplify algebraic expressions such as, $2x + 5x$ by combining like terms.

$$2x + 5x = 7x$$

We can also simplify algebraic expressions such as $\frac{2x}{5x}$ by removing any factors of 1.

$$\frac{2x}{5x} = \frac{2 \bullet x}{5 \bullet x} = \frac{2}{5}$$

Also, we can apply rules of signed numbers when simplifying algebraic expressions such as $-\frac{3}{4x} \bullet \frac{x}{3}$.

$$-\frac{3}{4x} \bullet \frac{x}{3} = -\frac{3 \bullet x}{4x \bullet 3} = -\frac{3 \bullet x}{4 \bullet 3 \bullet x} = -\frac{1}{4}$$

Demonstration Problems	*Practice Problems*
Simplify.	Simplify.
9. (a) $-\frac{3}{4} \bullet \frac{5x}{6x} =$	**9. (b)** $-\frac{2a}{3a} \bullet \frac{6}{5} =$
10. (a) $\frac{2}{5} \div \frac{p}{q} =$ ____ $\bullet$ ____ $=$ ____	**10. (b)** $\frac{3}{8} \div \frac{a}{b} =$ ____ $\bullet$ ____ $=$ ____
11. (a) $\left(-\frac{2}{5}\right) \div \left(-\frac{4}{x}\right) =$ ____ $\bullet$ ____ $=$ ____	**11. (b)** $\left(-\frac{3}{7}\right) \div \left(-\frac{12}{a}\right) =$ ____ $\bullet$ ____ $=$ ____
	Answers: **9. (b)** $-\frac{4}{5}$; **10. (b)** $\frac{3b}{8a}$; **11. (b)** $\frac{a}{28}$