
5.1 Introduction to Decimals

A *decimal* is a fraction whose denominator is a power of 10 and whose numerator is expressed by figures placed to the right of a decimal point. For example:

$$1.53 = 1 \text{ and } \frac{53}{100}$$

Why do we have decimals? Aren't fractions good enough? Let's explore making comparisons between fractions and decimals.

Which fraction is larger?

$$\frac{17}{20} \text{ or } \frac{21}{25}$$

With like denominators, we can write them as follows:

$$\frac{17 \cdot 5}{20 \cdot 5} = \frac{85}{100} \text{ and } \frac{21 \cdot 4}{25 \cdot 4} = \frac{84}{100}$$

Which is larger?

$$\frac{85}{100} \text{ or } \frac{84}{100}$$

Clearly, $\frac{85}{100}$ is larger than $\frac{84}{100}$.




Therefore, $\frac{17}{20}$ is larger than $\frac{21}{25}$.




Notice that $\frac{85}{100} = 0.85$ and $\frac{84}{100} = 0.84$.

The decimal form requires much less space and effort to write than the fractional form does. So, we could say decimals are a shorthand notation for decimals.

In general, comparing fractions with denominators that are powers of 10 is much easier than comparing fractions with different denominators. When quick comparison of two numbers is desired, we can write fractional portions as decimals. If we don't need a comparison of two numbers, then we might prefer to write them as fractions. Let's look at examples where each of the above might be the best choice.

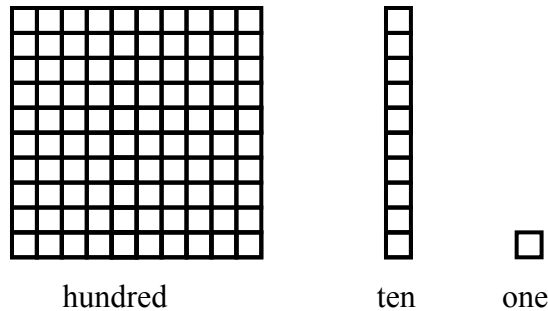
Checkmark the column for which is better for each of the following:

| | Decimals | Fractions |
|---|----------|-----------|
| Money  | | |
| Tape measure  | | |
| Recipes  | | |

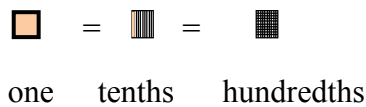
| | Decimals | Fractions |
|--|----------|-----------|
| Gas pumped into a tank  | | |
| Car gas gauge  | | |
| Winning times in Olympic events  | | |

Graphic Representation of Decimals

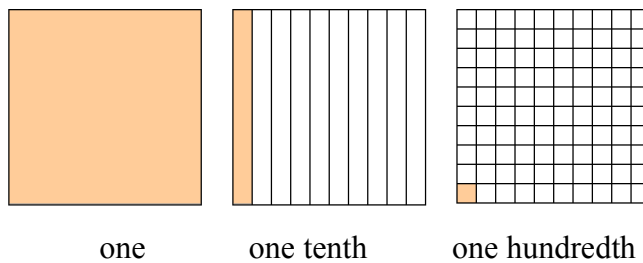
Recall that we can represent whole numbers graphically as follows:



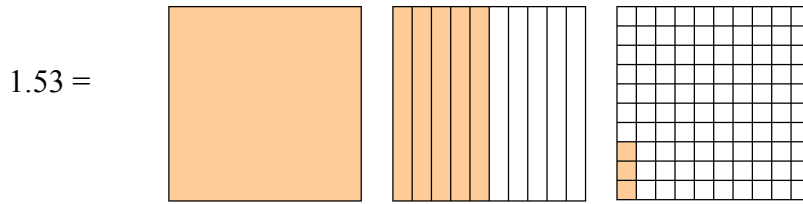
To represent decimals graphically, we must divide up the smallest square.



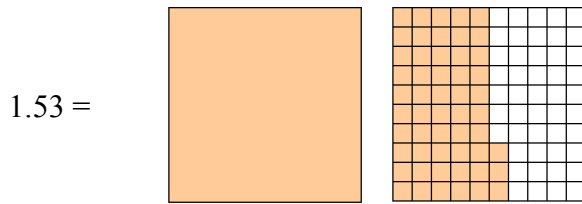
It appears that we will need to zoom in on “one” to be able to see the divisions.



We can now represent 1.53 graphically:



Or, alternatively:



| <i>Demonstration Problems</i> | <i>Practice Problems</i> |
|--|--|
| <p>Shade the diagram to represent the following numbers.</p> <p>1. (a) 1.41</p> <div style="display: flex; justify-content: space-around; align-items: center; margin: 10px 0;"> <div style="border: 1px solid black; width: 100px; height: 100px;"></div> <div style="border: 1px solid black; width: 100px; height: 100px; background-image: linear-gradient(to right, repeating-linear-gradient(4px, transparent, transparent 2px, black 2px, black 4px));"></div> </div> <p>2. (a) 0.88</p> <div style="display: flex; justify-content: center; align-items: center; margin: 10px 0;"> <div style="border: 1px solid black; width: 100px; height: 100px; background-image: linear-gradient(to right, repeating-linear-gradient(4px, transparent, transparent 2px, black 2px, black 4px));"></div> </div> | <p>Shade the diagram to represent the following numbers.</p> <p>1. (b) 1.35</p> <div style="display: flex; justify-content: space-around; align-items: center; margin: 10px 0;"> <div style="border: 1px solid black; width: 100px; height: 100px;"></div> <div style="border: 1px solid black; width: 100px; height: 100px; background-image: linear-gradient(to right, repeating-linear-gradient(4px, transparent, transparent 2px, black 2px, black 4px));"></div> </div> <p>2. (b) 0.22</p> <div style="display: flex; justify-content: center; align-items: center; margin: 10px 0;"> <div style="border: 1px solid black; width: 100px; height: 100px; background-image: linear-gradient(to right, repeating-linear-gradient(4px, transparent, transparent 2px, black 2px, black 4px));"></div> </div> |
| <p>Answers: 1. (b) ; 2. (b) </p> | |

Place Value

| | | | | | | | | | | | |
|-------------------|---------------|-----------|----------|------|------|---|--------|------------|-------------|-----------------|---------------------|
| Hundred-thousands | Ten-thousands | Thousands | Hundreds | Tens | Ones | . | Tenths | Hundredths | Thousandths | Ten-thousandths | Hundred-thousandths |
| | | 1 | 2 | 3 | 4 | . | 5 | 6 | 7 | 8 | 9 |

To name a decimal using words, use the following guidelines:

1. Name the part to the left of the decimal point the same way that was described in section 1.1.
2. The decimal point is read as “and”.
3. Name the part of the number to the right of the decimal point as if it were a whole number.
4. Name the decimal place of the right-most non-zero digit.

Here are some examples:

| | | | |
|--------|-----------------------|---------------------------|--------------------------|
| Number | 4.3 | 5.07 | 2.009 |
| Name | four and three tenths | five and seven hundredths | two and nine thousandths |

| | | |
|--------|---------------------------------|---|
| Number | 5.34 | 2.358 |
| Name | five and thirty-four hundredths | two and three hundred fifty-eight thousandths |

| | |
|--------|---|
| Number | 1234.56789 |
| Name | one thousand two hundred thirty-four and fifty-six thousand seven hundred eighty-nine hundred-thousandths |

| <i>Demonstration Problems</i> | <i>Practice Problems</i> |
|---|--|
| Write the word name for each decimal. 3. (a) 5.4 | Write the word name for each decimal. 3. (b) 3.8 |
| 4. (a) 10.32 | 4. (b) 12.59 |
| 5. (a) 3.017 | 5. (b) 4.012 |
| Write the number for each decimal named. 6. (a) nine and four tenths | Write the number for each decimal named. 6. (b) three and two tenths |
| 7. (a) twelve and five hundredths | 7. (b) eleven and eight hundredths |
| 8. (a) forty-two and forty-two thousandths | 8. (b) fifty-one and thirteen thousandths |
| Answers: 3. (b) three and eight tenths; 4. (b) twelve and fifty-nine hundredths; 5. (b) four and twelve thousandths; 6. (b) 3.2; 7. (b) 11.08; 8. (b) 51.013 | |

| <i>Demonstration Problems</i> | <i>Practice Problems</i> |
|---|---|
| Convert each decimal to a fraction or mixed number. 9. (a) 5.8 10. (a) 0.34 11. (a) 0.008 Place the correct symbol between each pair <, >, or =. 12. (a) 3.4 _____ 1.98 13. (a) 5.823 _____ 5.8223 14. (a) -0.15 _____ 0.15 | Convert each decimal to a fraction or mixed number. 9. (b) 2.9 10. (b) 0.48 11. (b) 0.001 Place the correct symbol between each pair <, >, or =. 12. (b) 4.3 _____ 1.72 13. (b) 5.412 _____ 5.4112 14. (b) -0.34 _____ 0.34 |
| Answers: 9. (b) $2\frac{9}{10}$; 10. (b) $\frac{12}{25}$; 11. (b) $\frac{1}{1000}$; 12. (b) >; 13. (b) >; 14. (b) < | |

| <i>Demonstration Problems</i> | <i>Practice Problems</i> |
|---|--|
| <p>15. (a) Round 3.796 to the nearest tenth.</p> | <p>15. (b) Round 2.154 to the nearest tenth.</p> |
| <p>16. (a) Round 15.264 to the nearest hundredth.</p> | <p>16. (b) Round 46.582 to the nearest hundredth.</p> |
| <p>17. (a) Round 198.2787 to the nearest thousandth.</p> | <p>16. (b) Round 596.3726 to the nearest thousandth.</p> |
| <p>Round the following to the nearest (i) tenth, (ii) hundredth, and (iii) thousandth.</p> | <p>Round the following to the nearest (i) tenth, (ii) hundredth, and (iii) thousandth.</p> |
| <p>18. (a) 12.7981</p> | <p>18. (b) 15.4952</p> |
| <p>(i)</p> | <p>(i)</p> |
| <p>(ii)</p> | <p>(ii)</p> |
| <p>(iii)</p> | <p>(iii)</p> |
| <p>Answers: 15. (b) 2.2; 16. (b) 46.58; 17. (b) 596.374; 18. (b) (i) 15.5, (ii) 15.50, (iii) 15.495</p> | |