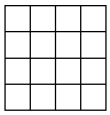
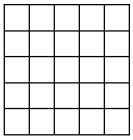
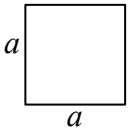
5.7 Simplify and Use Square Roots

The ***square*** of a number is the value of that number multiplied by itself.
For example, $5^2 = 5 \times 5 = 25$.
Why is this exponent referred to as a square? Study the following examples.

Number	Geometric Square	Algebraic Square
1		$1^2 = 1 \times 1 = 1$
2		$2^2 = 2 \times 2 = 4$
3		$3^2 = 3 \times 3 = 9$
4		$4^2 = 4 \times 4 = 16$
5		$5^2 = 5 \times 5 = 25$

Notice that the side of each square is the same as the given number and the area of each square is the same as the square of the given number.



Area = a^2

Suppose we are given a number that is a square of a number. Can we find the number that was squared? The process is called finding the ***square root***. We use the notation $\sqrt{\ }$.

Square	Square Root
$6^2 = 6 \times 6 = 36$	$\sqrt{36} = \sqrt{6^2} = 6$
$7^2 = 7 \times 7 = 49$	$\sqrt{49} = \sqrt{7^2} = 7$
$8^2 = 8 \times 8 = 64$	$\sqrt{64} = \sqrt{8^2} = 8$
$9^2 = 9 \times 9 = 81$	$\sqrt{81} = \sqrt{9^2} = 9$
$10^2 = 10 \times 10 = 100$	$\sqrt{100} = \sqrt{10^2} = 10$
$a^2 = a \times a$	$\sqrt{a^2} = a$ (for $a > 0$)

Demonstration Problems	*Practice Problems*
Evaluate the following square roots.	Evaluate the following square roots.
1. (a) $\sqrt{64}$	**1. (b)** $\sqrt{49}$
2. (a) $\sqrt{16}$	**2. (b)** $\sqrt{9}$
3. (a) $\sqrt{81}$	**3. (b)** $\sqrt{100}$
4. (a) $\sqrt{4}$	**4. (b)** $\sqrt{1}$
5. (a) $\sqrt{36}$	**5. (b)** $\sqrt{25}$
	Answers: **1. (b)** 7; **2. (b)** 3**; 3. (b)** 10; **4. (b)** 1; **5. (b)** 5

Recall that the product of two negative numbers is positive.

negative × *negative* = *positive*

Square	Square
$6^2 = 6 \times 6 = 36$	$(-6)^2 = -6 \times -6 = 36$
$7^2 = 7 \times 7 = 49$	$(-7)^2 = -7 \times -7 = 49$
$8^2 = 8 \times 8 = 64$	$(-8)^2 = -8 \times -8 = 64$
$9^2 = 9 \times 9 = 81$	$(-9)^2 = -9 \times -9 = 81$
$10^2 = 10 \times 10 = 100$	$(-10)^2 = -10 \times -10 = 100$
$a^2 = a \times a$	$\sqrt{a^2} = a$ (for $a > 0$)

For every square number, there are two distinct numbers whose square is that number. Since we don't want to have two different values resulting from an operation, then we will always choose the principal square root, defined as follows:

Definition	Example
For any real number a, the ***principal square root*** of a^2 is $\sqrt{a^2} = \lvert a \rvert$	If $a = -5$, then $\sqrt{a^2} = \sqrt{(-5)^2} = \lvert -5 \rvert = 5$

It should be noted that the square root of a negative number is not a real number. For example, $\sqrt{-9}$ is not a real number because $3^2 = 9$ and $(-3)^2 = 9$.

Demonstration Problems	*Practice Problems*
Evaluate the following square roots.	Evaluate the following square roots.
6. (a) $\sqrt{(-3)^2}$	**6. (b)** $\sqrt{(-8)^2}$
7. (a) $\sqrt{-9}$	**7. (b)** $\sqrt{-16}$
8. (a) $\sqrt{7^2}$	**8. (b)** $\sqrt{2^2}$
	Answers: **6. (b)** 8; **7. (b)** not a real number; **8. (b)** 2

To find the square root of a large number, we can find the prime factorization of the number and analyze its factors.

For example, to evaluate $\sqrt{384}$, find the prime factorization of 384.

$$384 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$

Let's rearrange these factors as

$$2 \cdot 3 \cdot 3 \cdot 2 \cdot 3 \cdot 3$$

Notice that

$$2 \cdot 3 \cdot 3 \cdot 2 \cdot 3 \cdot 3 = 18 \cdot 18$$

This tells us that $\sqrt{384} = 18$.

Some numbers are not square numbers and do not have whole number square roots. However, we can approximate their square roots as follows.

For example, to approximate $\sqrt{90}$, we locate the position of 90 relative to consecutive square numbers.

Square
$1^2 = 1 \times 1 = \mathbf{1}$
$2^2 = 2 \times 2 = \mathbf{4}$
$3^2 = 3 \times 3 = \mathbf{9}$
$4^2 = 4 \times 4 = \mathbf{16}$
$5^2 = 5 \times 5 = \mathbf{25}$
$6^2 = 6 \times 6 = \mathbf{36}$
$7^2 = 7 \times 7 = \mathbf{49}$
$8^2 = 8 \times 8 = \mathbf{64}$
$9^2 = 9 \times 9 = \mathbf{81}$
$10^2 = 10 \times 10 = \mathbf{100}$

Since

$$9^2 = 81 < 90 < 100 = 10^2$$

then

$$9 < \sqrt{90} < 10$$

$$\sqrt{90} \approx 9.5$$

With a calculator we can get a better approximation, but this method gives a general idea of the value of certain square roots.

Demonstration Problems	*Practice Problems*
Evaluate the following square roots.	Evaluate the following square roots.
9. (a) $\sqrt{196}$	**9. (b)** $\sqrt{225}$
10. (a) $-\sqrt{441}$	**10. (b)** $-\sqrt{400}$
11. (a) $\sqrt{-441}$	**11. (b)** $\sqrt{-400}$
12. (a) $\sqrt{25+144}$	**12. (b)** $\sqrt{9+16}$
13. (a) $\sqrt{25}+\sqrt{144}$	**13. (b)** $\sqrt{9}+\sqrt{16}$
Approximate the following square root.	Approximate the following square root.
14. (a) $\sqrt{40}$	**14. (b)** $\sqrt{30}$
Answers: **9. (b)** 15; **10. (b)** –20;	**11. (b)** not a real number; **12. (b)** 5; **13. (b)** 7; **14. (b)** ≈ 5.5