
8.2 Solving Equations Using the Division and Multiplication Properties of Equality

Properties of Equality
Division Property of Equality

For any nonzero real number c , if $a = b$, then

$$\frac{a}{c} = \frac{b}{c}$$

Multiplication Property of Equality

For any nonzero real number c , if $a = b$, then

$$ac = bc$$

We'll first look at how we can use the Division Property of Equality. This property allows us to “undo” a multiplication that was done to the variable in an equation.

Example (a) Solve

$$3x = 12$$

The division property of equality will preserve the equality when we divide both sides of the equation by 3.

$$\begin{aligned} 3x &= 12 \\ \frac{3x}{3} &= \frac{12}{3} \\ 1 \cdot x &= 4 \\ x &= 4 \end{aligned}$$

Alternatively, we can multiply both sides of the equation by $\frac{1}{3}$.

$$\begin{aligned} 3x &= 12 \\ \frac{1}{3} \cdot 3x &= \frac{1}{3} \cdot 12 \\ 1 \cdot x &= 4 \\ x &= 4 \end{aligned}$$

To check the solution, we replace x with 4 in the original equation.

$$\begin{aligned} 3x &= 12 \\ 3 \cdot 4 &= 12 \quad ? \\ 12 &= 12 \quad \text{Yes} \end{aligned}$$

The solution set is $\{4\}$.

Next, we will use the Multiplication Property of Equality. This allows us to “undo” a division that was done to the variable.

Example (b) Solve

$$\frac{x}{5} = 7$$

We can multiply both sides of the equation by 5, (the reciprocal of $\frac{1}{5}$)

$$\begin{aligned} \frac{x}{5} &= 7 \\ \frac{1}{5}x &= 7 \\ 5 \cdot \frac{1}{5}x &= 5 \cdot 7 \\ 1 \cdot x &= 35 \\ x &= 35 \end{aligned}$$

To check the solution, we replace x with 35 in the original equation.

$$\begin{aligned} \frac{x}{5} &= 7 \\ \frac{35}{5} &= 7 \quad ? \\ 7 &= 7 \quad \text{Yes} \end{aligned}$$

The solution set is $\{35\}$.

Example (c) Solve $\frac{2}{5}x = 6$

We can preserve equality by multiplying both sides of the equation by the reciprocal of $\frac{2}{5}$, which is $\frac{5}{2}$.

$$\begin{aligned}\frac{2}{5}x &= 6 \\ \frac{5}{2} \cdot \frac{2}{5}x &= \frac{5}{2} \cdot 6 \\ 1 \cdot x &= 15 \\ x &= 15\end{aligned}$$

To check the solution, we replace x with 15 in the original equation.

$$\begin{aligned}\frac{2}{5}x &= 6 \\ \frac{2}{5} \cdot 15 &= 6 \text{ ?} \\ 6 &= 6 \\ \text{Yes}\end{aligned}$$

Alternatively, we could divide both sides of the equation by $\frac{2}{5}$, although it is not an efficient method for solving this particular equation.

$$\begin{aligned}\frac{2}{5}x &= 6 \\ \frac{\frac{5}{2}}{\frac{2}{5}} &= \frac{6}{\frac{2}{5}} \\ 1 \cdot x &= 6 \div \frac{2}{5} \\ x &= 6 \cdot \frac{5}{2} \\ x &= 15\end{aligned}$$

The solution set is $\{15\}$.

<i>Demonstration Problems</i>	<i>Practice Problems</i>
Solve and check. 1. (a) $-5w = 30$ 2. (a) $\frac{4}{5}m = 40$	Solve and check. 1. (b) $-3x = 21$ 2. (b) $\frac{3}{7}y = 9$
Answers: 5. (b) -7 ; 6. (b) 21	