
8.4 Solving Equations with Fraction or Decimal Coefficients

Equations Containing Fractions

When equations contain fractions of different denominators, the steps for solving can become lengthy. We can shorten the process by eliminating the fractions using the multiplication property of equality.

For example, consider $y + \frac{1}{3} = \frac{2}{5}$

Instead of adding $-1/3$ to both sides of the equation, we can multiply both sides of the equation by the least common denominator of the two fractions, 15, as follows

$$\begin{array}{rcl}
 y + \frac{1}{3} & = & \frac{2}{5} \\
 15(y + \frac{1}{3}) & = & 15(\frac{2}{5}) \\
 15 \cdot y + 15 \cdot \frac{1}{3} & = & 15 \cdot \frac{2}{5} \\
 15y + 5 & = & 6 \\
 \underline{-5 \quad -5} & & \\
 15y + 0 & = & 1 \\
 \frac{15y}{15} & = & \frac{1}{15} \\
 y & = & \frac{1}{15}
 \end{array}$$

$$\begin{array}{rcl}
 \text{Check:} & y + \frac{1}{3} & = \frac{2}{5} \\
 & \frac{1}{15} + \frac{1}{3} & = \frac{2}{5} \quad ? \\
 & \frac{1}{15} + \frac{5}{15} & = \frac{2}{5} \quad ? \\
 & \frac{6}{15} & = \frac{2}{5}
 \end{array}$$

$$\text{Solution set: } \left\{ \frac{1}{15} \right\}$$

<i>Demonstration Problems</i>	<i>Practice Problems</i>
Solve and check. 1. (a) $\frac{1}{2}y - 1 = \frac{1}{3}y - 1$ Check:	Solve and check. 1. (b) $\frac{1}{3}p - 5 = \frac{1}{4}p$ Check:
Answers: 1. (b) 60	

Equations Containing Decimals

Consider the equation $y = 0.2y + 0.72$

Compare the following two methods for solving this equation:

Method 1:

$$\begin{aligned}
 y &= 0.2y + 0.72 \\
 1.0y &= 0.2y + 0.72 \\
 \underline{-0.2y \quad -0.2y} & \\
 0.8y &= 0 + 0.72 \\
 \frac{0.8y}{0.8} &= \frac{0.72}{0.8} \\
 y &= 0.9
 \end{aligned}$$

Method 2:

$$\begin{aligned}
 y &= 0.2y + 0.72 \\
 100y &= 100(0.2y + 0.72) \\
 100y &= 100 \cdot 0.2y + 100 \cdot 0.72 \\
 100y &= 20y + 72 \\
 \underline{-20y \quad -20y} & \\
 80y &= 0 + 72 \\
 \frac{80y}{80} &= \frac{72}{80} \\
 y &= \frac{9}{10}
 \end{aligned}$$

Method 2 includes extra steps, but eliminates tedious decimal operations. To use method 2 on equations that contain decimals, multiply both sides of the equation by a power of 10 with the same number of zeroes as the greatest number of decimal places appearing in the equation.

<i>Demonstration Problems</i>	<i>Practice Problems</i>
Solve and check. 2. (a) $0.1a - 0.35 = 0.2a - 8.3$ Check:	Solve and check. 2. (b) $0.02x - 1.56 = 0.8x$ Check:
Answers: 2. (b) -2	

Mixed Practice

<i>Demonstration Problems</i>	<i>Practice Problems</i>
Solve and check. 3. (a) $\frac{1}{3}x + \frac{2}{3} + 5 = \frac{1}{4}x - \frac{1}{4}$	Solve and check. 3. (b) $\frac{1}{4}x + \frac{3}{4} + 2 = \frac{1}{3}x - \frac{1}{3}$
4. (a) $\frac{3}{2}x - \frac{9}{4} = 2x - \frac{1}{8}$	4. (b) $\frac{8}{3}x - \frac{10}{3} = 3x - \frac{5}{6}$
5. (a) $4.72 - 2.5x + 1.3 = 6.02$	5. (b) $5.34 - 0.5x + 2.06 = 7.2$
Answers: 3. (b) 37; 4. (b) $-\frac{15}{2}$; 5. (b) 0.4	